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Old and New Growth Theory: The Impact of Taxation

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This note provides a short introduction into the basic mechanism of growth models illustrating the effect of taxation on growth. An emphasis is put on the distinction between traditional growth models and new growth models. Given the macroeconomic perspective, taxation will be treated in a very simple way as a tax on total value added. No distinction can be made (due to space constraints) between taxation on different income sources or types of expenditure and incentive effects are only briefly mentioned. Distributional issues, though crucial for policy decisions, also need to be left in the background.

The model by Solow (1956) with exogenous technological progress is presented first. It is shown that taxation has an effect on the level of GDP only. The first new growth model, provided by Romer (1986), is presented next. It turns out that taxation influences not only the level of GDP but also its growth rate. The third model presented here, going back to Barro (1990), shows under which circumstances a public sector is required for an economy to grow. The optimal size of the public sector is derived. Finally, a brief outlook to current research, including the debate on scale-effects and taxation and growth, is given.

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[1] I am grateful to Jürgen Kröger, Werner Schüle and Sjak Smulders for comments.

1 The traditional growth model

1.1 The model

An economy is best described by its technology, its factor endowment and by preferences of households. We first present such a model economy, first described by Solow (1956). The next section will investigate the growth process of such an economy.

- Technologies

The technology is of the Cobb-Douglas type

$$Y(t) = AK(t)^{\alpha} L^{1-\alpha}, \tag{1}$$

where A is total factor productivity, $K(t)$ is the capital stock, L is labour and α is the output elasticity of capital. This technology is used for production of both consumption goods and investment goods.

The change $\dot{K}$ in the capital stock of the economy is given by the difference between net investment I and depreciation δK,

$$\dot{K} = I - \delta K. \tag{2}$$

The dot on K describes how K changes over time, i.e. the derivative with respect to time, $\dot{K} = dK/dt$.

- Factor endowment

Factors of production are capital K and labour L. The stock of labour is considered to be fixed (which is not crucial for results, however). The initial capital stock, i.e. the capital stock when the analysis of the economy starts, is given by K_0. Subsequently, the capital stock evolves as a function of investment and depreciation (2).

- Preferences

Households decide how much of total output they want to consume. The remainder is used for consumption. We assume that saving decisions of households imply an aggregate investment of

$$I = sY, \tag{3}$$

where s is the aggregate constant saving rate.

1.2 The evolution of capital

The evolution of capital in this economy directly follows from (2). Inserting the production function (1) into the investment equation (3) and the investment equation (3) into (2), we get

$$\dot{K} = sAK^{\alpha}L^{1-\alpha} - \delta K. \tag{4}$$

As by assumption all variables other than K are constant, this equation unambiguously describes the evolution of capital. Figure 1 illustrates this.

The horizontal axis shows the capital stock K, the vertical axis its change $\dot{K}$ in time. The dashed lines show gross investment I and depreciation δK. Summing them up results in the solid line. If this line is above zero, $\dot{K}$ is positive and capital is accumulated. Taking an example economy which at some point in time t_0 (which might be the year of industrial revolution in this country after which capital could be accumulated) had a capital stock of K_0, then the capital stock of this economy increases continuously over time. This is indicated by the arrow ▶ on the horizontal axis.

The longer accumulation of capital continues, the closer the economy is at its steady state K^*. This capital stock is characterized by equality between gross investment and depreciation. Hence, with (4),

$$\dot{K} = 0 \Rightarrow sA\left(L/K^*\right)^{1-\alpha} = \delta. \qquad (5)$$

Should an economy ever have a capital stock exceeding K^*, the capital stock would decrease and the economy would approach its steady state K^* from above, as indicated by the arrow ◄.

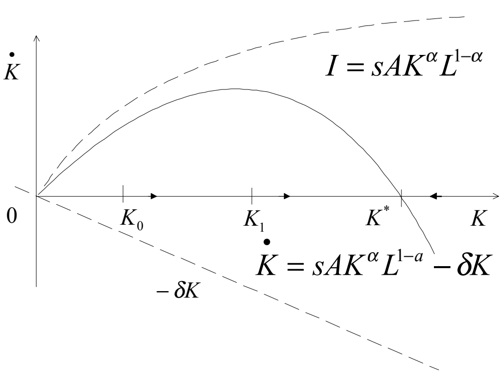

Figure 1: The evolution of the capital stock

1.3 The effects of taxation

Let us now introduce taxation. Let capital income and labour income be taxed at the same rate ψ. This reduces disposable income by $1-\psi$ and we obtain instead of (3) a new investment equation

$$I = s\left(1-\psi\right)Y.$$

Tax income is used by the state to provide some public good. This public good has no direct productive effect. This assumption is made by analytical convenience and should empirically probably not be true. Due to the assumption of fixed supply of labour, taxation of (labour) income has no distortionary effect. The implication of both assumptions will be discussed below.

Capital accumulation (4) then follows

$$\dot{K} = s\left(1-\psi\right)AK^{\alpha}L^{1-\alpha} - \delta K.$$

This implies that the steady state level (5) reduces to

$$s\left(1-\psi\right)A\left(L/K^*\right)^{1-\alpha} = \delta.$$

As long-run growth is not affected by taxation in this theoretical setup, taxation has a level effect but not a growth effect. This is illustrated in the following figure.

In an economy with a tax rate "tax1", GDP per worker Y/L increases at a constant rate which is exogeneously given. This is captured by the straight line in the figure where the logarithm of GDP per worker is plotted over time. Now increase the tax rate to "tax2". This leads to a reduction in the steady state level of capital as just shown. GDP per worker at each time is lower in the high-tax economy. The growth rate, however, is - by construction of the model - unaffected.

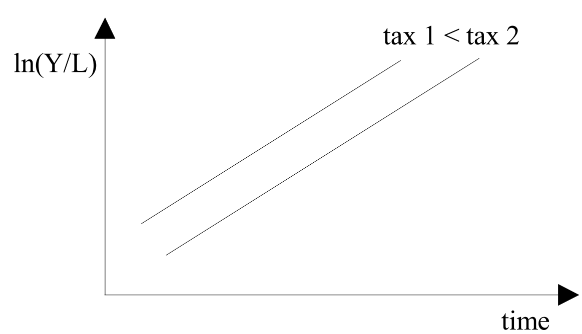

Figure 2: Taxation and the level of GDP per capita

If labour supply was endogenous, taxation would reduce labour supply which would further increase the level difference between countries with high and low taxes.

If taxes are used to provide a public good that is an input in production (e.g. public infrastructure), there is an optimal level of taxation. A country with higher taxes could actually have a higher GDP per worker than a country with lower taxes, reversing the results illustrated in the above figure.

Even when GDP per worker is lower, it should be kept in mind that this does *not* necessarily mean lower welfare. When taxes are levied in order to provide public good that is valued by inhabitants of a country (like clean local environment) and to the extent that higher taxes mean higher provision of the public good, a society with higher taxes and therefore lower output can be better off in terms of the welfare. Clearly, there is an optimal level of taxation. A more elaborated model would be needed in order to analyse this trade-off. The next section contains important features of such an analysis.

2 New growth theories I: Externalities

Let us now study a model which illustrates the idea of an endogenous explanation of economic growth. Such an explanation would allow to understand where differences in the growth rate come from. With exogenous technological progress postulated in the Solow type growth theory such an understanding is impossible for the long-run. This section presents a simplified version (a constant saving rate is assumed) of the model developed by Romer (1986).

2.1 The model

- The basic idea

The basic idea in Romer's model is the external effect of capital (cf. Romer, 1986). Capital is understood not only as the physical production unit but also as knowledge required to run this production unit. This knowledge is not only available in the firm that currently uses this production unit but also externally. Examples include the producers of this production unit or potential customers which however decided not to buy it. Another reason for general availability of this knowledge is that it might be applicable also to similar types of production units. Knowledge that is used in one firm therefore partially has the character of a public good.

Installing a production unit, i.e. accumulating capital, does therefore not only increase the capital stock in this firm but also knowledge in this firm and, more importantly, knowledge in the economy as a whole. This basic idea can be captured by the technology

$$y = Ak^{\alpha}l^{1-\alpha}f(K), \quad \text{where } K = \Sigma k.$$

The consumption and investment good y is produced by using capital k and labour l with total factor productivity being given by A. In addition to the usual Cobb-Douglas-structure, the term $f(K)$ captures the knowledge-externality of capital. This externality is by assumption positive, $f'(K) > 0$. More private capital k in one firm therefore increases knowledge K in all firms.[1]

The effect of a firm's capital stock on overall knowledge is neglected by each individual firm. Firms therefore consider the externality $f(K)$ as exogenous and treat it just like

[1] Easterly (2001, ch. 8) provides nice examples for this externality facilitating e.g. the rapid growth of the Bangladesh garment industry in the 1980s.

total factor productivity A. Production in a firm is therefore characterized by constant returns to scale and firms continue to produce under perfect competition.

Aggregating over all firms gives an economy-wide production function

$$Y = AK^{\alpha}L^{1-\alpha}f(K). \tag{6}$$

The model can be completed by the usual equation describing capital accumulation

$$\dot{K} = sY - \delta K. \tag{7}$$

What is now the growth rate for capital, GDP and GDP per capita in an economy described by (6) and (7)?

- A simpler model

Letting analytical problems not become overwhelming, we assume a simple expression for the externality

$$f(K) = K^{\beta},\ \beta > 0. \tag{8}$$

Inserting this expression into the technology (6) and the resulting expression in the capital accumulation equation (7), we get

$$\dot{K} = sAK^{\alpha+\beta}L^{1-\alpha} - \delta K. \tag{9}$$

Capital grows therefore at the rate

$$\frac{\dot{K}}{K} = sAK^{\alpha+\beta-1}L^{1-\alpha} - \delta.$$

The stronger the externality, i.e. the higher the parameter β, the higher the growth rate. For the special case where $\beta = 1 - \alpha$, the growth rate is constant and given by

$$g = sAL^{1-\alpha} - \delta. \tag{10}$$

2.2 Properties of the model

2.2.1 Differences to the Solow growth model

The endogeneity of the growth rate is the first crucial difference to the model of Solow. Despite constant total factor productivity and constant population size, the capital stock grows in the long-run, provided of course a sufficiently large externality,

$$\beta \geq 1 - \alpha.$$

The second main difference consists in the impact of saving rates and population size: Whereas the Solow model predicts that a higher saving rate increases GDP per capita but does not affect the growth rate, this model predicts that a higher saving rate implies a higher rate of growth.

2.2.2 The role of decreasing returns to capital

The neoclassical model of growth cannot explain long-run growth of GDP per capita without technological progress. The origin of this property are decreasing returns to the capital as can be seen in equation (4) or figure 1. As marginal returns decrease, the curve for k decreases. An additional unit of capital does increase GDP and, given the constant saving rate, this increases investment. At some point, however, this increase no longer compensates linear depreciation.

From a modelling perspective, the trick of the new growth theory consists in avoiding in one way or another decreasing returns to capital. Various channels can be imagined. The model by Romer presented above assumes economy-wide externalities which imply constant returns to capital at the aggregate level but decreasing returns to capital at the firm level. Lucas (1988) adds accumulation of human capital to capital accumulation and Romer (1990) introduces R&D with knowledge spillovers which implies an increasing total factor productivity.

2.2.3 Growth without convergence

Comparing two economies modeled as above, it is easy to understand why convergence of GDP per capita does not take place. Given the special case $\beta = 1 - \alpha$ for the externality (8), the technology (6) reads

$$Y = AK^{\alpha}L^{1-\alpha}K^{1-\alpha} = AKL^{1-\alpha}.$$

A constant growth rate (10) for capital is an immediate implication.

The third crucial difference to the model of Solow is therefore the absence of any convergence. Independently of the initial capital stock K_0 (or capital stock per capita) the growth rate of the economy is immediately given by the constant in (10). GDP therefore also increases instantaneously, without any adjustment dynamics, at the constant rate $\dot{Y}/Y = \dot{K}/K = g$. As a consequence, no convergence among countries takes place.

2.2.4 Determinants of growth

As no convergence takes place, GDP per capita in various countries will not grow at the same growth rate, neither in the short run, nor in the long-run. At what rate do countries grow then?

Equation (10) directly provides determinants of growth. The higher the saving rate s of a country, the higher total factor productivity A, the higher the growth rate. When depreciation δ is stronger, the growth rate is lower.

Population size L has a positiv impact on the growth rate as well. China or India should have a higher growth rate than Singapore or Hong Kong. This is a property of these models which was often criticised (Jones, 1995). Various authours (cf. outlook below) have presented more elaborate growth models that solve this problem. A simple reinterpretation of population size, however, would also be sufficient. If population size is not simply the number of inhabitants but the aggregate human capital stock of inhabitants, a country that is larger in terms of population is no longer predicted to grow faster. The model by Romer (1990) has the feature that the aggregate human capital stock is a central determinant of growth.

It is important to keep in mind that growth does not only refer to growth of capital or GDP but also to growth of GDP per capita. Long-run growth of GDP per capita does no longer depend on exogenous technological progress as in the old growth theory but on features that are inherent to an economy. The long-run growth rate is endogenously determined.

2.3 The effects of taxation

2.3.1 Growth effects

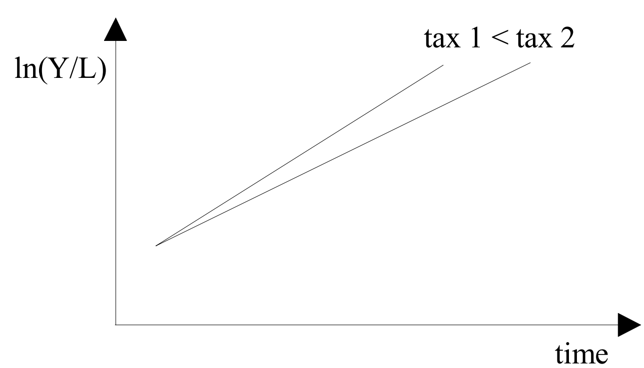

Figure 3: Taxation and growth of GDP per capita

We now introduce taxes in this setup as well. With an identical tax rate ψ on capital and labour income again, the equation for capital accumulation (9) is modified to

$$\dot{K} = s(1-\psi) AK^{\alpha+\beta} L^{1-\alpha} - \delta K.$$

The growth rate in such an economy now reads for $\beta = 1-\alpha$

$$g = s(1-\psi) AL^{1-\alpha} - \delta.$$

The impact of taxation on GDP per worker is illustrated in figure 3. Again, the logarithm of GDP per worker is plotted over time. Economies that start with the same GDP per worker diverge over time. The economy with lower taxes grows faster.

2.3.2 Welfare effects

Here, as well, it is important to keep in mind that taxes might be used for productive purposes. To this extent, differences in growth rates of GDP per capita across countries do *not* necessarily reflect differences in welfare across individuals of these countries. This is illustrated by the following model.

Let welfare of households be determined by private consumption $C(\tau)$ and the provision of a public good $G(\tau)$. Preferences are therefore described by

$$U(t) = \int_t^\infty e^{-\rho(\tau-t)} u(C(\tau), G(\tau))\, d\tau \quad (11)$$

where $u(.)$ is the instantaneous utility function, ρ is the time preference rate, t is the date of today and the planning horizon is infinity (∞). Instantaneous utility increases in private consumption $C(\tau)$ and the public good $G(\tau)$. The latter could be thought of as clean environment. The present value of future instantaneous discounted utility is denoted by $U(t)$. When the saving rate is given by s and all tax income is used for the provision of the public good, private consumption and public provision of the good is given by

$$C(t) = s[1-\psi] Y(t), \quad G(t) = \psi Y(t)$$

where $Y(t)$ is GDP before tax. Making standard assumptions concerning the instantaneous utility function $u(.)$ (i.e. assuming homothetic preferences), instantaneous utility is a function of the tax rate and linear in GDP

$$u(C(\tau), G(\tau)) = u(s[1-\psi], \psi) Y(t).$$

Letting GDP grow at a constant rate that decreases in the tax rate as was found above, $Y_\tau = Y_t e^{\alpha g(1-\psi)[\tau-t]}$, welfare (11) can be written after some steps as

$$U(t) = u(s[1-\psi], \psi) Y_t \frac{1}{\rho - \alpha g(1-\psi)}. \quad (12)$$

where $\left[e^{(\alpha g(1-\psi)-\rho)[\tau-t]}\right]_t^\infty = -1$ was used as $\alpha g(1-\psi) - \rho < 0$ (this last condition needs to hold such that $U(t)$ is finite and can be maximized).

The expression in (12) shows the importance of distinguishing between growth and welfare. While the growth rate clearly decreases when the tax rate increases (in this setup, cf. the discussion on extensions below), effects on welfare $U(t)$ depend on three channels: (i) $U(t)$ clearly decreases through the growth effect $1/(\rho - \alpha g(1-\psi))$. A higher tax rate decreases the growth rate which makes the denominator $(\rho - \alpha g(1-\psi))$ larger and thereby decreases welfare. (ii) A higher growth rate also decreases private consumption as income after taxation is lower, the higher the tax rate. This is captured by the term $s[1-\psi]$. (iii) Finally, however, the tax rate increases the provision of the public good which increases welfare (after all, this is the objective of the provision of the public good in this setup). This is the second term ψ in the instantaneous utility function $u(s[1-\psi],\psi)$. Maximisation of (12) with respect to the tax rate ψ would yield an optimal tax rate and an optimal provision of the public good.

For identical preferences, the same optimal tax rate would result for each country. It is an empirical issue (not necessarily a straightforward one) to find out to what extent international differences in taxation can be traced back to international differences in preferences. If differences in preferences can be found, differences in taxation lead to differences in growth rates that do not necessarily imply differences in welfare. Some countries might simply be happier by growing faster and other countries are happier by having more public goods.

3 New growth theories II: The public sector

Technological differences and differences in preferences can explain cross-country differences in growth. Economic policy, however, can also easily be imagined to have an impact on long-run growth. The next section presents a very easy model that can be used in the subsequent section to analyse the growth effects of economic policy.

3.1 The "AK" model

The simplest model generating positive long-run growth was developed by Rebelo (1991). He assumes a production function of the type $Y = AK$ (giving the model its name) and thereby simply postulates constant returns to capital. (The original paper also has a more complex specification.) In such a setup, accumulation of capital follows, given an accumulation equation like (2) and a constant saving rate,

$$\dot{K} = sAK - \delta K. \tag{13}$$

The growth rate of capital is then straightforwardly

$$\frac{\dot{K}}{K} = sA - \delta.$$

Due to constant returns to capital, the growth rate is independent of the current capital stock. If the saving rate is sufficiently high or depreciation is sufficiently low, the capital stock, GDP and GDP per capita instantaneously and forever grow at a constant rate,

$$\dot{Y}/Y = \dot{K}/K = sA - \delta.$$

3.2 Optimal saving

It is useful for the subsequent analysis to allow for optimal saving behaviour of households. We therefore depart from the constant saving rate and assume that households maximise a utility function $U(t)$ of the type

$$U(t) = \int_t^{\infty} e^{-\rho(\tau - t)} u(C(\tau))\, d\tau \tag{14}$$

where ρ is the time preference rate and instantaneous utility $u(C(\tau))$ is given by

$$u(C(\tau)) = C(\tau)^{1-\sigma}.$$

Maximisation is achieved by optimally choosing the time path $C(\tau)$ of consumption (which implicitly gives an optimal saving rate). The resource constraints of all households is

$$\dot{K} = AK - C - \delta K.$$

The saving rate s in (13) has been replaced by the difference between output and consumption.

Optimal consumption implies the Keynes-Ramsey rule (for details cf. textbooks on growth or lecture notes available from the author)

$$\sigma \frac{\dot{C}}{C} = A - \delta - \rho.$$

Consumption is therefore growing at the constant rate

$$g = \frac{A - \delta - \rho}{\sigma}.$$

The capital stock, GDP and GDP per capita grow at the same rate. It is positive if total factor productivity A is high enough or depreciation and the time preference rate are sufficiently low. The reason for positive long-run growth are again constant returns to capital.

3.3 A model with a public sector

This model will now be used to study the impact of fiscal policy on growth (Barro, 1990).

- Technologies

The technology used for producing consumption and investment goods is characterised by a total factor productivity that depends on the provision of a rivalrous public good G. One can think of property rights, rule of law or safety in public spaces as such a public good,

$$Y = A \left[\frac{G}{Y} \right]^{\beta} K. \tag{15}$$

As a total factor productivity is given by $A\left[G/Y\right]^{\beta}$, the basic underlying structure of the technology is of the AK-type. The public good G is rivalrous which is captured by dividing G by Y. The output elasticity of the public good is denoted by β.

- The government

The government levies indirect taxes at a tax rate ψ on all goods. Tax revenue is used to provide the public good G,

$$G = \psi Y. \tag{16}$$

- Households

Households obtain capital and labour income, pay taxes, save and consume what is left over. Aggregating over households gives an aggregate increase of the capital stock of

$$\dot{K} = Y - G - \delta K - C = (1 - \psi) Y - \delta K - C. \tag{17}$$

Maximisation of the utility function (14) results in optimal consumption following

$$\sigma \frac{\dot{C}}{C} = (1 - \psi) \frac{\partial Y}{\partial K} - \delta - \rho. \tag{18}$$

- The growth rate of consumption

Inserting the budget constraint (16) of the government into the technology (15) of firms, this technology becomes

$$Y = A \psi^{\beta} K.$$

Marginal productivity of capital is therefore $\partial Y/\partial K = A\psi^{\beta}$ and dependent on fiscal policy as captured by the tax rate ψ. The growth rate of consumption (18) is then

$$\sigma\frac{\dot{C}}{C} = (1-\psi)\, A\psi^{\beta} - \delta - \rho. \qquad (19)$$

3.4 The effects of taxation

Apart from the usual determinants of growth like total factor productivity A, depreciation δ or saving behaviour, captured here implicitly by the time preference rate ρ, fiscal policy has an impact on growth as well via the tax rate ψ. Fiscal policy has an impact through two channels: on the one hand, indirect taxes decrease the growth rate of consumption. This effect is captured by $(1-\psi)$ and describes the reduction of capital and labour income in the budget constraint (17) of households. This reduction reduces interest rates for capital income in the consumption equation (18). This reduces saving incentives.

On the other hand, indirect taxes ψ increase the growth rate of consumption as a higher tax rate allows a larger provision of the public good G which in turn increases returns to capital accumulation. This effect is captured by ψ^{β}. Both effects jointly lead to a non-monotonic relationship between growth and tax policy.

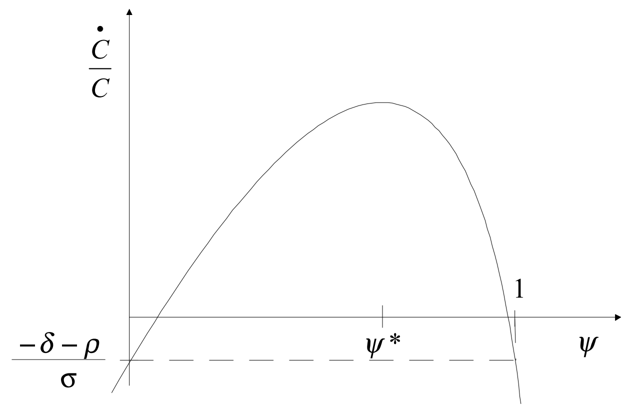

Figure 4: Taxation and growth of consumption

As illustrated by this figure, the highest growth rate is obtained at

$$\psi^* = \frac{\beta}{1+\beta}$$

which can be easily seen by maximising (19) with respect to the tax rate.

4 Outlook

As briefly mentioned previously, the finding of early endogenous growth models that larger countries grow faster has been subject to various criticism (Jones, 1995). Alternative models have been presented (Klundert and Smulders, 1995; Segerstrom, 1998; Young, 1998; Howitt, 1999; Peretto and Smulders, 2002) where scale effects are (almost) absent. A discussion of these issues and an overview is provided by Jones (1999).

Taxation has been treated in a very simple fashion in order to illustrate some basic mechanisms. These results are far from being comprehensive and policy conclusions can not be drawn from these simple calculations.

Understanding the underlying market structure and the types of market failures inherent in any economy is crucial for judging in a particular case whether taxes imply higher or lower growth or welfare. A recent contribution that studies growth effects of taxation in a model without scale effects is by Zeng and Zhang (forthcoming).

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