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Production technologies in stochastic continuous time models

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ABSTRACT

Properties of dynamic stochastic general equilibrium models can be revealed by either using numerical solutions or qualitative analysis. Very precise and intuition-building results are obtained by working with models which provide closed-form solutions. Closed-form solutions are known for a large class of models some of which, however, have some undesirable features such as potentially negative output. This paper offers closed-form solutions for models which are just as tractable but do not suffer from these shortcomings.

1. Introduction

Continuous time models with uncertainty are widely used in economics. The seminal papers are the finance papers by Merton (1969, 1971) and his (1975) Solow growth model with stochastic population growth. These methods were used subsequently – for what could be called an early endogenous growth model – by Eaton (1981). Eaton introduced uncertainty in the production process, highlighting technological uncertainty. Versions of his formulation of the production process, which we will call "differential" in what follows, were used by many authors in several fields. Recent examples include Asea and Turnovsky (1998), Bianconi and Turnovsky (2005), Chatterjee et al. (2004), Chattopadhyay and Turnovsky (2004), Clemens and Soretz (2004), Corsetti (1997), Epaulard and Pommeret (2003), Evans and Kenc (2003), García-Peñalosa and Turnovsky (2005), Gokan (2002), Gong and Zou (2002, 2003), Grinols and Turnovsky (1993, 1994, 1998a, b), Kenc (2004), Pommeret and Smith (2005), Smith (1996), Steger (2005), Turnovsky (1993, 1999a, b, 2000), Turnovsky and Smith (2006) and Koethenbuerger and Lockwood (2010).

While apparently very inspiring and useful for understanding a wide range of issues, the differential production technology has at least two major shortcomings. First, modeling the production process by specifying a differential instead of a standard production function is intuitively hard to understand. Similar difficulties arise when trying to interpret the goods market equilibrium condition embedded in a setup with a differential formulation of the production process. Second, as already Grinols and Turnovsky (1998a) have noticed in footnote 4 of their paper "[…] the flow of output […] may become

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negative." They suggest, however, that "[…] the advantages to computation and modeling are sufficiently great to justify this specification, despite this unappealing aspect."[1] Nevertheless, this implication of this modeling choice as well as its difficult interpretation might be the reasons why continuous time methods under uncertainty are not as widely used in macroeconomics as they could be and as these methods would deserve, given their convenient properties.

It is the purpose of this paper to show that an alternative formulation of production technologies is available which does not have the shortcomings of the differential approach. To this end, Section 2.1 presents the basic demand side consisting of an agent that maximizes a standard utility function. Section 2.2 then presents the differential representation of technologies while Section 2.3 offers the alternative formulation. Section 3.1 then generalizes existing closed-form solutions for comparison purposes.

Our main theoretical contributions are all in the subsequent Section 3.2 which models technologies by standard production functions. We first present a closed-form solution of a maximization problem with logarithmic instantaneous utility and both one Wiener process and many Poisson processes as the source of uncertainty for the evolution of total factor productivity (TFP). The technology is *AK*.

We then present closed-form solutions for maximization problems where the instantaneous utility function is different from the logarithmic specification but still belongs to the constant relative risk aversion (CRRA) class. For these non-logarithmic cases, uncertainty affects the accumulation technology of the economy via a stochastic depreciation process. The fundamental source of uncertainty is the same combination of a Wiener process and many Poisson processes as previously. The technology is again of the *AK* nature. One can read this section as a reinterpretation of the resource constraint with differential formulations of the technology such that the formulation of the technology is standard. In this reading, our contribution is this reinterpretation and the generalization of results to a setup with a Wiener process and many Poisson processes.

Individual components of our contributions have been used in the literature. Cagetti et al. (2002) use a special case of the formulation of TFP as we do for the logarithmic utility function case. They do not provide closed-form solutions, however. Rebelo and Xie (1999) provide closed-form solutions for a case which is a special case of our specification with stochastic depreciation. They do this for the *AK* case and with a parametric restriction for the general Cobb–Douglas case. They do not stress the equivalence to the solution of models using the differential representation of technologies, however. There is a large literature that works with models which exhibit closed-form solutions if the parameter set of the model is restricted. Probably the first contribution of this type can be traced back to Merton (1975, p. 388). Recent examples include Wälde (2005), Smith (2007) and Posch (2009). We will return to this approach in the conclusion.

2. The model

2.1. The planner

We study a classic saving-investment problem. A social planner chooses the amount of consumption $C(\tau)$ so as to maximize a social welfare function representing expected utility of infinitely lived households,

$$U_t = E_t\left[\int_t^\infty e^{-\rho[\tau-t]}u(C(\tau))\,d\tau\right], \tag{1}$$

where ρ denotes the time preference rate. The instantaneous utility function is of the CRRA class,

$$u(C(\tau)) = \begin{cases} \dfrac{C(\tau)^{1-\gamma}-1}{1-\gamma} & \text{for } \gamma \neq 1, \\ \log C(\tau) & \text{for } \gamma = 1. \end{cases} \tag{2}$$

We see this central planner view as a short-cut to a decentralized economy without any distortions. All of the results which will follow below will therefore also hold in an economy with many households.

2.2. Differential representation of the technology

This section presents the approach which specifies technologies by a differential. In order to be comparable as much as possible to the approach appearing in Section 2.3.2 below, we modify the differential setup, as it can be found in the literature, in two ways. First, we make the depreciation rate δ explicit. The depreciation rate is usually implicitly contained in the constant production technology factor A. We replace A by $A-\delta$ which is of course just a notational matter and does not alter any of the implications of the differential model. Second, instead of adding a stochastic component in the equation for the production technology we subtract this term. Again, this is just a notational matter and only changes the signs of the corresponding parameters of the stochastic processes considered. It will, however, be advantageous later when comparing the approach we suggest with this differential approach.

[1] Other authors are aware of this as well. See e.g. footnote 5 in Monfort and Pommeret (2002) or Smith (2007, Section 6). Smith also points out that problems of this type also appear in the precautionary savings literature.

The production process of an economy can now be specified – following a large part of the literature as cited in the introduction – by

$$dY(t) = (A-\delta)K(t)\,dt - K(t)\,dw(t), \tag{3}$$

where $Y(t)$ is aggregate output in t, $(A-\delta)$ is a constant measure of the TFP less the deterministic depreciation rate δ, $K(t)$ is the capital stock, and the increment $dw(t)$ will be specified below. The flow of output is used for capital accumulation and for consumption, modeled as

$$dY(t) = dK(t) + dC(t). \tag{4}$$

Combining Eqs. (3) and (4) gives the expression for the capital accumulation which is equal to investment[2]

$$dK(t) = ((A-\delta)K(t) - C(t))\,dt - K(t)\,dw(t). \tag{5}$$

This and similar processes are used for example by Evans and Kenc (2003), Gertler and Grinols (1982), Gokan (2002), Gong and Zou (2002, 2003), Grinols and Turnovsky (1998a, b), Steger (2005), or Turnovsky (1993). In most of these papers the stochastic increment in Eq. (3) is defined to be $dw(t) = \sigma dz(t)$ where $dz(t)$ is a Brownian motion and the σ is some variance measure of output.

While some authors acknowledge (see e.g. footnote 4 in Grinols and Turnovsky, 1998a) the fact that this implies the possibility of a negative Y, we briefly recapitulate this property here by simply solving the differential Eq. (3) starting in $t=0$ with $Y(0) \equiv Y_0$ and $z(0) \equiv 0$,

$$Y(t) = Y_0 + (A-\delta)\int_0^t K(s)\,ds - \sigma \int_0^t K(s)\,dz(s).$$

To see that this is in fact a solution of the above flow-representation just apply Ito's Lemma and recover (3).[3] This says that with a constant factor input K, i.e. $Y(t) = Y_0 + (A-\delta)Kt - \sigma K z(t)$, output in t is determined by a deterministic and a stochastic part. The latter one is often interpreted to reflect various random elements affecting output production (e.g. Gong and Zou, 2003). Ignoring Y_0, the deterministic part $(A-\delta)t$ implies linear growth (i.e. not exponential growth as is usually the case and empirically more relevant) and the stochastic part $\sigma z(t)$ implies deviations from the trend. As $z(t)$ is Brownian motion, the sum of the deterministic and the stochastic part can become negative.

The unusual formulation in (3) and the property that $Y(t)$ can become negative might be two reasons why this approach is not as popular as it could be. We will now present standard formulations of technologies which make models just as tractable as with this formulation.

2.3. Standard representation of the technology

We now present technologies in the standard way. We use a typical AK structure and introduce uncertainty either through uncertainty in TFP – "standard I" – or through uncertainty in the capital accumulation process – "standard II."

2.3.1. Standard I: stochastic TFP

This section will now preserve the spirit of the technology used above – the AK-structure – but expresses the technology in a way such that it has standard properties. The economy produces the final good Y according to

$$Y(t) = A(t)K(t). \tag{6}$$

Capital is the only factor of production and TFP is given by $A(t)$. It follows the geometric stochastic process

$$dA(t) = \mu A(t)\,dt + A(t)\,dw(t), \tag{7}$$

where the increment $dw(t)$ in its most general formulation will be specified below as a combination of one Wiener and many Poisson processes.[4]

Capital accumulation is equal to investments $I(t) = Y(t) - C(t)$ net of depreciation,

$$dK(t) = (Y(t) - \delta K(t) - C(t))\,dt. \tag{8}$$

2.3.2. Standard II: stochastic depreciation

There is an alternative where output is left in the standard formulation (6), TFP remains constant at A, i.e. $dA(t)=0$, and hence (7) is not valid. In contrast, following Rebelo and Xie (1999), we assume that capital accumulation is stochastic. Let there be a standard goods market clearing condition $Y(t) = C(t) + I(t)$ but let capital accumulation be risky, e.g. by a stochastic

[2] This Eq. (5) can be obtained under the additional assumption that $dC(t) = C(t)\,dt$ where "…consumption over the instant dt occurs at the non-stochastic rate $C(t)\,dt$." (cf. Turnovsky and Smith, 2006, on p. 251). The expression $dC(t) = C(t)\,dt$ can be found expicitly e.g. in Grinols and Turnovsky (1998a, on p. 503) or in Turnovsky (2000, on p. 400) and is at odds with the prediction of this model (see further below) that optimal consumption is a constant fraction ψ of the capital stock, i.e. $C(t) = \psi K(t)$, making consumption stochastic and not just a function $C(t)$ times the deterministic time differential dt.

[3] For an introduction to the methods behind this and for more background, see Wälde (2009, part IV). Chang (2004) devotes an entire chapter (Chapter 5) on closed-form solutions for Brownian motion.

[4] Special cases of this general expression have been used in the literature. See e.g. Cagetti et al. (2002) for the Brownian motion case with drift.

depreciation process:

$$dK(t) = I(t)\,dt - K(t)(\delta\,dt + dw(t)).$$

Again, $w(t)$ is a combination of a Wiener and Poisson processes. This capital accumulation equation gives, together with the goods market clearing condition and the production technology, an expression of

$$dK(t) = (AK(t) - \delta K(t) - C(t))\,dt - K(t)\,dw(t). \tag{9}$$

Notice that this equation is identical to the capital accumulation process of the differential setup in (5). Hence, if $w(t)$ is a simple Brownian motion we have a similar formulation of the resource constraint as Eaton (1981) and others, only that we have obtained it with a more standard setup with standard output and a stochastic capital accumulation process.

The interpretation that the stochastic part of the capital accumulation process is the result of stochastic depreciation was already suggested by Eaton (1981, footnote 2). In the differential approach, stochastic capital accumulation is the *result* of combining the assumptions about the stochastic evolution of output in (3) and the goods market clearing condition in (4). By contrast, in the standard II approach proposed here, the fact that capital accumulation is risky is an *assumption* which results in a stochastic evolution of output. This has the advantage of explicitly specifying where uncertainty comes from and avoids specifications of output which can easily be criticized. With more conventional technologies as proposed here, the continuous time approach to uncertainty could become more popular.

3. Closed-form solutions

It is well-known that closed-form solutions exist for a variety of models. Table 1 summarizes the structure of the models presented above and provides a preview of the optimal consumption levels. This section generalizes existing closed-form solutions and compares them to the new ones—which are based on models with standard production functions.

The differential approach widely used in the literature and its features are presented in the second column. Approaches with standard production functions appear in columns three and four. The notation used in this Table is as in the text. The only new parameter is

$$\psi \equiv \frac{\rho}{\gamma} - \frac{1-\gamma}{\gamma}(A-\delta) + (1-\gamma)\frac{1}{2}\sigma^2 - \frac{1}{\gamma}\sum_{i=1}^{n}\lambda_i((1-\beta_i)^{1-\gamma}-1). \tag{10}$$

Following the idea of Merton (1976), we allow the stochastic component $w(t)$ in the technology (column 2), in TFP (column 3) or in depreciation (column 4) to be a mixture of both a continuous and countably many jump processes, $w(t) = \sigma z(t) + \sum_{i=1}^{n} \beta_i q_i(t)$, implying that

$$dw(t) = \sigma dz(t) + \sum_{i=1}^{n} \beta_i dq_i(t). \tag{11}$$

The differential $dz(t)$ stands for the increment of standard Brownian motion, i.e. $z(t) \sim N(0,t)$, whereas the $dq_i(t)$ for $i=1,\ldots,n$ describe increments of independent Poisson processes with arrival rates λ_i. By setting $\beta_i = 0$ the formulation used by Eaton (1981) and in the bulk of the literature thereafter is obtained. With $\sigma = 0$, it collapses to a pure jump setup.

3.1. Differential approach

The definition for uncertainty in (11) applied to the capital accumulation process in (5) yields

$$dK(t) = ((A-\delta)K(t) - C(t))\,dt - \sigma K(t)\,dz(t) - \sum_{i=1}^{n} \beta_i K(t)\,dq_i(t). \tag{12}$$

Table 1
Model overview and preview of results.

	Differential (stoch. output)	Standard I (stoch. TFP)	Standard II (stoch. depreciation)
output	$dY = (A-\delta)Kdt - Kdw$	$Y = AK$	$Y = AK$
TFP	constant	$\frac{dA}{A} = \mu dt + dw$	constant
capital	$dK = dY - dC$	$dK = Idt - \delta Kdt$	$dK = Idt - \delta Kdt - Kdw$
investment	$Idt = dK$	$I = Y - C$	$I = Y - C$
stoch. process	$dw = \sigma dz + \sum_{i=1}^{n} \beta_i dq_i$	identical	identical
log-utility	$C = \rho K$	$C = \rho K$	$C = \rho K$
CES-utility	$C = \psi K$	n.a.	$C = \psi K$

In order to obtain a closed-form solution, one needs to solve the following nonlinear ordinary differential equation called the maximized Bellman equation,

$$\rho V(K) = \log\left(\frac{1}{V_K}\right) + V_K\left((A-\delta)K - \frac{1}{V_K}\right) + \frac{1}{2}\sigma^2 K^2 V_{KK} + \sum_{i=1}^{n} \lambda_i (V(K-\beta_i K) - V(K)). \tag{13}$$

Solving requires an educated guess about the value function V whose structure can be shown to be identical to that one of the well-known case where $\beta_i = 0\ \forall i$ (cf. e.g. Merton, 1969), i.e.

$$V(K) \equiv \frac{\log(\psi K)}{\psi} + \Psi \tag{14}$$

with

$$\psi = \rho, \quad \Psi = \frac{A-\delta-\rho-\frac{1}{2}\sigma^2 + \sum_{i=1}^{n}\lambda_i \log(1-\beta_i)}{\rho^2}.$$

This explicit form of the value function leads to the conclusion that optimal consumption is just the constant fraction ρ of the capital stock,

$$C^*(t) = \rho K(t). \tag{15}$$

The result that C is a constant fraction of K is robust to allowing a more general instantaneous utility function as in (2). This implies optimal consumption to be given by

$$C^*(t) = \psi K(t), \tag{16}$$

where the constant ψ is the one from (10).

3.2. Standard approaches

3.2.1. Standard I

In contrast to the differential approach, we now let TFP, $A(t)$, to be a function of time and stochastic disturbances. Allowing $\mu \neq 0$ in the formulation for the TFP process (cf. Eq. (7) with (11)), we specify the parameters μ, σ, β_i, and the arrival rates λ_i of the Poisson processes q_i such that $A(t)$ is a martingale, i.e. the expectation about the TFP level at some future point in time is given by the current level, $E_t[A(\tau)] = A(t)$ for $\tau \geq t$, implying that $A(t)$ has no trend. Given the AK technology in (6), this assumption is made in order to preserve a stationary expected growth rate of the economy which is governed by the marginal product of capital $A(t)$. As $E_t[A(\tau)] = A(t)e^{(\mu + \sum_{i=1}^{n}\beta_i\lambda_i)(\tau-t)}$ the restriction of no trend in the TFP-process requires that

$$\mu + \sum_{i=1}^{n} \beta_i \lambda_i = 0. \tag{17}$$

Economically speaking, given that the arrival rates λ_i are positive by construction, technological progress is either technological regress ($\mu < 0$) or there are occasional negative shocks, i.e. $\beta_i < 0$. We find the second interpretation more plausible where oil price shocks, natural or other disasters disrupt the smooth evolution of an economy. But it should be kept in mind that without an AK specification, one could allow for both positive μ and β_i.

With this setup and a logarithmic utility, the maximized Bellman equation is a second-order nonlinear partial differential equation,[5]

$$\rho V(A,K) = \log\left(\frac{1}{V_K}\right) + V_K\left(AK - \delta K - \frac{1}{V_K}\right) + V_A A\mu + \frac{1}{2} V_{AA} A^2 \sigma^2 + \sum_{i=1}^{n} \lambda_i (V(A + A\beta_i, K) - V(A,K)). \tag{18}$$

It is standard in the literature to provide an "educated guess" and prove a verification theorem. Finding such a guess can sometimes be very time-consuming. For our case here in (18), we can be more systematic and derive a solution.

Use the definition of the value function as the expected utility out of current and all future optimal consumption, $V_t(A,K) \equiv \max_{\{C(\tau)\}} U_t$ subject to constraints where U_t is the objective function from (1). Looking at the results from the literature as well as from the previous section, a plausible guess concerning optimal consumption is

$$C^*(t) \equiv \tilde{\psi} K(t), \tag{19}$$

where the $\tilde{\psi}$ is an arbitrary constant. Inserting this conjecture into the objective function we can analyze whether the resulting integral can be solved. In fact, doing so by deriving expressions for $K(t)$ from (8) and for $A(t)$ from (7) with (11), respectively, together with the guess (19) gives

$$V_t(A,K) = E_t\left[\int_t^\infty e^{-\rho(\tau-t)} \log(\tilde{\psi} K(t) e^{\int_t^\tau (A(t)e^{\varpi} - \delta - \psi)\, ds}) d\tau\right], \tag{20}$$

[5] For a proof that dynamic programming methods can be used with unbounded instantaneous utility functions, see Sennewald (2007). Applications are in Sennewald and Wälde (2006).

where

$$\varpi=\mu(s-t)-\frac{1}{2}\sigma^2(s-t)+\sigma z(s)+\log\left(\sum_{i=1}^{n}\frac{1+\beta_i}{n}\right)\sum_{i=1}^{n}\frac{q_i(s)}{n}.$$

Simplifying and solving this integral and dropping the time subscript (as this expression holds for each point in time) yields the value function,

$$V=\frac{\log(\tilde{\psi}K)}{\rho}+\frac{A}{\rho^2}-\frac{\delta+\tilde{\psi}}{\rho^2}. \tag{21}$$

It is easy to verify that, for $\tilde{\psi}=\rho$, this is indeed a solution to the maximized Bellman equation. Therefore it is confirmed that optimal consumption is a constant fraction of the capital stock,

$$C^*(t)=\rho K(t). \tag{22}$$

Unfortunately unlike in the case with the differential technology this result holds only for the logarithmic utility (2) and not for the CRRA case of $\sigma\neq 1$.

3.2.2. Standard II with Brownian motion and many poisson processes

Fortunately, however, closed from solution can be found for CRRA utility functions with $\sigma\neq 1$ and standard technologies. The alternative to stochastic TFP we propose is a stochastic capital accumulation technology via e.g. stochastic depreciation. Let the $dw(t)$ in (9) again stand for a combination of a Wiener process and many Poisson processes as in (11):

$$dK(t)=(AK(t)-C(t))\,dt-K(t)\left(\delta\,dt+\sigma\,dz(t)+\sum_{i=1}^{n}\beta_i dq_i(t)\right). \tag{23}$$

Of course this equation is identical to the corresponding expression in the differential setup in (12). Therefore, computing the solution for the maximization problem of the social planner both in the case with log-utility (2) and in the case with the more general instantaneous utility function (2) leads to the same rule concerning optimal consumption, i.e. consumption is just the constant fraction ρ (cf. Eq. (15)) and ψ (cf. Eq. (16)), respectively, of the capital stock.

4. Conclusion

The differential approach to specifying technologies is widely used. Specifications of technologies of this type in continuous time models with uncertainty have undesirable features such as potentially negative output. These features might be one reason why modeling uncertainty in continuous time is not as popular in macroeconomics as it could be. One major advantage of these specifications is their analytical tractability.

This paper generalizes existing closed-form solutions for differential technologies. More importantly, it proposes two alternative setups where technologies are modeled by standard production functions. In both cases, the undesirable features of the differential approach are not present and the analytical tractability is preserved. Closed-form solutions are provided for both alternatives.

One of these alternatives implies solutions which are identical to solutions of models following the differential approach. This shows that all results obtained so far in the literature can be preserved even without the undesirable features. For all future papers, analytical tractability can be obtained just as easily as before.

One should not conceal, however, that closed-form solutions obtained here have their shortcomings as well. If optimal consumption is proportional to capital (as in Table 1) but independent of total factor productivity, properties of optimal consumption are bound to be empirically questionable. The way out of this dilemma – analytically tractable closed-form solutions on the one hand and empirical relevance on the other – seems to be provided by closed-form solutions for models with parameter restrictions. In these models (see the introduction for some references), output is always positive and optimal consumption is a function of total factor productivity as well. The standard case there is the one where a constant savings rate is optimal. So maybe one should say good-bye to *AK* models altogether and rater accept explicit parameter restrictions in models with a richer production structure.

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