*The Economic Journal*, **109** (*March*), C156–C171.  Published by Blackwell Publishers, 108 Cowley Road, Oxford OX4 1JF, UK and 350 Main Street, Malden, MA 02148, USA.

# A MODEL OF CREATIVE DESTRUCTION WITH UNDIVERSIFIABLE RISK AND OPTIMISING HOUSEHOLDS*

*Klaus Wälde*

This paper studies optimal household behaviour in a model of creative destruction. The saving technology is characterised by stochastic returns that follow a Poisson process. It is shown that equilibrium conditions with optimising households differ substantially from equilibrium conditions where investment in R&D is determined by firms. Three out of four market failures disappear and a new market failure resulting from a complementarity in financing R&D is identified. Studying the social optimum shows that it contains as the special case of risk neutrality the social optimum derived in the literature.

A remarkable model in the growth literature was developed by Aghion and Howitt (1992). They stress the creative destruction aspect of technological change by studying the implications of the introduction of a new technology which renders an old technology obsolete. The only form of investment in their economy consists of an R&D process with uncertain outcome where the probability of research success depends on the amount of investment into R&D. Aghion and Howitt assume that individuals have utility functions linear in consumption and can therefore neglect almost all aspects related to optimal household behaviour. They assume either the existence of a frictionless Walrasian capital market where savings receive a certain return $r$ or the absence of a capital market and individuals who consume all their income at every moment in time.

Assuming utility functions linear in consumption is very restrictive, however. This paper introduces a CES utility function and explicitly studies optimal behaviour of households in Aghion and Howitt's framework. Though this extensions might appear minor, it does have considerable consequences. Optimal saving behaviour has to take jumps in, *inter alia*, total factor productivity and factor income into consideration. The source of these jumps is the Poisson process that governs R&D ventures. This implies aggregate undiversifiable risk and discontinuities in most economic variables. The solution method required is therefore different from Aghion and Howitt's approach.

Some equilibrium properties of the Aghion and Howitt model are preserved, others can be shown to be a special case of the model presented here while still others change completely. It continues to be true that the expected equilibrium growth rate increases as e.g. the total factor productivity parameter $\gamma$ increases or as the time preference rate falls. The social optimum of Aghion

* This paper was written while working at the University of Dortmund. Discussions with Gerhard Glomm, Wolfgang Leininger and Jan Wilmanns and financial support by the Graduiertenkolleg 'Allokationstheorie, Wirtschaftspolitik und kollektive Entscheidungen' of the Deutsche Forschungsgemeinschaft are gratefully acknowledged. Views expressed in this paper do not necessarily represent the views of the World Bank, its Executive Directors, or the countries they represent.

and Howitt is the special case of risk neutrality of the planner's solution derived here. In contrast to their model, the decentralised economy is dominated by a strategic complementarity between individual and aggregate investment. Analysing the social planner's solution shows that this complementarity *ceteris paribus* implies overinvestment.

The strategic complementarity between individual and aggregate investment arises since one R&D project is financed by several households. When many investors jointly finance a research project, gains from this project have to be divided among investors. The share in profits of the next monopolist therefore depends on investment behaviour of others which creates an externality. Joint financing always takes place when households are risk averse. In the Aghion and Howitt framework, risk neutral firms run independent R&D projects and no division of gains is necessary.

A second market failure, called 'appropriability effect' and identified by Aghion and Howitt in their model, is present here as well. It tends to reduce aggregate investment. The remaining three market failures identified by Aghion and Howitt are all internalised by optimising households. In a representative agent model as used here, all households own some share in the current and in the upcoming monopolist. Households therefore internalise the 'business stealing' effect caused by replacing the old monopolist and 'correctly' discount future expected payoffs. They also internalise the product market distortion effect caused by the monopolist.

This paper also contributes to the literature on optimal saving under uncertainty and presents – to the best of the author's knowledge – the first study of optimal saving in a general equilibrium context when the underlying stochastic variables are Poisson distributed. Optimal control of diffusion processes was introduced by Merton (1969, 1971, 1990), who also performed a partial equilibrium optimal control analysis of Poisson processes. Growth models were analysed by Bourguignon (1974), Merton (1975), Bismut (1975) and Eaton (1981). More recently, Turnovsky (1993) has studied the effects of monetary and tax policy in an uncertain world, where uncertainty stems from stochastic processes generated by Brownian motion. Obstfeld (1994) has studied the growth increasing effects of increasing international risk-sharing. Poisson processes and creative destruction effects, however, have not been studied.[1]

## 1. The Model

### 1.1. *Production Side*

The model developed by Aghion and Howitt (1992) consists of three sectors and the summary here will be brief. In the first, the consumption good $y$ is

[1] For an introduction to continuous time methods under uncertainty cf. Dixit and Pindyck (1994), Turnovsky (1995) and Chow (1979).

produced by many price taking firms according to a Cobb-Douglas type technology,

$$y = \gamma^t x^\alpha, \qquad \alpha < 1. \tag{1}$$

The productivity in the consumption good sector, $\gamma^t$, is determined by the currently most advanced technology $t$. The invention of each new technology rises $t$ by one and therefore total factor productivity by $\gamma > 1$. Each firm employs intermediate goods $x$ and some indivisible factor of production, normalised to unity. Firms maximise profits $\pi_y = y - px$ by choosing the optimal amount of intermediate goods,

$$x = \left(\gamma^t \alpha p^{-1}\right)^{1/(1-\alpha)}. \tag{2}$$

Firms take both the price of the consumption good (chosen as numeraire and normalised to unity) and the price of the intermediate good $p$ as given. This first order condition implies that profits of a firm in the consumption good sector amount to

$$\pi_y = (1-\alpha)\gamma^t x^\alpha. \tag{3}$$

The second sector is dominated by a monopolist who offers intermediate goods $x$ to the final good sector. Her technology is given by

$$x = L, \tag{4}$$

where $L$ denotes employment of labour in sector $x$. Given the monopolist's demand function (2) and profit function $\pi_x = px - wx$, the optimal price is set as a constant mark-up over wages $w$,

$$p = w/\alpha, \tag{5}$$

and her profits amount to

$$\pi_x = \alpha \pi_y. \tag{6}$$

Finally, there is an R&D sector, where research for new technologies is undertaken. This research is risky and does not necessarily lead to a successful end. The arrival rate of this research process is given by

$$\Gamma = \lambda n, \tag{7}$$

where $\lambda$ is a constant and $n$ stands for the amount of labour employed in the research sector. Firms in this sector produce under perfect competition and sell shares $\chi$ to households. Each share binds the firm to employ $\varphi$ workers for doing research. The 'production function' for a share therefore reads $\chi = \varphi^{-1} n$ and profits of an R&D firm are given by $\pi_n = \omega\chi - wn$, where $\omega$ is the *ex-ante* value of one share. Firms take prices of the share and labour costs as given and maximise (deterministic) profits by choosing employment $n$. This implies $\omega\varphi^{-1} = w$ and that total receipts $I = \omega\chi$ from selling these shares are used by firms to pay for labour costs,

$$I = wn. \tag{8}$$

When the research project is successful, the successful firm will use the new technology $t+1$ to drive the old monopolist out of the intermediate good market and will then produce the intermediate good itself. Profits made in the intermediate good sector are then distributed among investors who had financed the successful research project. The *ex-post* value of a share is zero when research is not successful.

Labour is the only factor that is mobile between sectors. Total supply is $N$ and the wage rate is determined on the labour market which is assumed to clear at every moment in time,

$$n + L = N. \tag{9}$$

## 1.2. *Consumption Side*

The uncertainty resulting from the R&D sector implies that households take consumption and investment decisions in an uncertain environment. Their utility at a point in (continuous) time $\tau$,

$$U(\tau) = \mathrm{E}\int_{\tau}^{\infty} e^{-\rho(1-\tau)}\, u[c(l)]\, dl, \tag{10}$$

is given by the expected discounted sum of future instantaneous utility $u(\cdot)$ which results from consumption flows $c(\cdot)$. The time preference rate is given by $\rho$. We will assume throughout the paper that instantaneous utility is given by

$$u(c) = c^{\sigma}. \tag{11}$$

These preferences imply an intertemporal elasticity of substitution of $\varepsilon = (1-\sigma)^{-1}$ and a constant relative risk aversion of $\varepsilon^{-1}$.[2]

A household chooses its optimal consumption level subject to a budget constraint which equates consumption expenditure $c$ plus investment $i$ to labour income $w$ and capital income $\pi$,

$$c + i = w + \pi. \tag{12}$$

Capital income results from shares held in firms active in the consumption and in the intermediate good sector. The only way households can invest in this economy is by buying shares in R&D projects. Aggregate investment $I$ therefore equals total receipts of R&D firms from selling their shares. When a household has financed a successful research project it acquires the right to a certain share $s$ of profits the successful firm earns in the intermediate good sector. We assume that a household receives a share of total profits that is given by her investment $i$ relative to total investment $I$ made into the same successful research project, $s = iI^{-1} \leqslant 1$. A successful research project also implies that

[2] Duffie and Epstein (1992) have introduced recursive utility functions that allow us to disentangle the elasticity of substitution and risk aversion in a continuous time setting with Brownian motion. Since the applicability of their approach to an environment with a Poisson process has not yet been fully worked out, we have to stick to the more traditional representation of preferences where implications of these two characteristics can not be distinguished.

the old monopolist is driven out of the market and that all shares held by a household in the old monopolist lose their value. When research projects are not successful, the amount of shares owned by the household does not change.[3] We capture this link between success of research projects and shares by a simple stochastic differential equation that reads

$$ds = \left(\frac{i}{I} - s\right) dq. \tag{13}$$

The change in shares $ds$ is a function of the increment $dq$ of a stochastic process $q$. Consistent with the formulation of the R&D sector, we assume that $q$ follows a Poisson process. The probability in a short time interval $dt$ that $dq = 1$ is given by $\Gamma dt$ and the probability that $dq = 0$ is given by $1 - \Gamma dt$, where $\Gamma$ is the arrival rate of the R&D process (7).[4] When a household does not invest in the upcoming vintage, its share holdings are reduced to zero $(-s)$ in case of a research success $(dq = 1)$. If it invests, the amount of share holdings depends on relative investment.

A household maximises its utility function (10), given instantaneous preferences (11) subject to (13) and (12) by choosing consumption. The Bellman equation reads (cf. Dixit and Pindyck, 1994)

$$\rho V(s, t) = \max_c \{u(c) + \Gamma[V(iI^{-1}, t+1) - V(s, t)]\}. \tag{14}$$

The value of receiving a share $s$ of profits of the monopolist having discovered technology $t$ is denoted by $V(s, t)$ and the value of receiving a share $iI^{-1}$ of the next monopolist's profits is denoted by $V(iI^{-1}, t+1)$. The first order condition is

$$u'(c) + \left[V\left(iI^{-1}, t+1\right) - V(s, t)\right] \frac{d}{dc}\Gamma + \Gamma \frac{d}{dc}\left[V\left(iI^{-1}, t+1\right) - V(s, t)\right] = 0. \tag{15}$$

Marginal utility from consumption, the gain from the new technology $t+1$ times the marginal arrival rate and the expected marginal gain from the next technology must add up to zero.

Following the idea of Merton (1969, 1971), one can find a closed form solution to the household's maximisation problem. Merton's idea consisted in guessing consumption and value functions that satisfy the first order condition and the Bellman equation. Value functions that sometimes fulfil these conditions generally have the same functional form as the instantaneous utility function $u(\cdot)$. Following this avenue, we guess that optimal consumption is a share $\delta$ out of current income,

$$c = \delta[w + \pi(s)], \tag{16}$$

where $\delta$ does not depend on household specific variables like e.g. wage income

[3] If households were allowed to trade shares (Wälde, 1998, c 7), they would not want to do so as long as they are identical in preferences and wealth.

[4] For more details on Poisson processes see for example Ross (1993).

and that the value function is of the form $V = \vartheta(w+\pi)^{\sigma}$, where $\vartheta$ is a constant. Inserting these guesses into (14) and (15) allows to solve for $\delta$. Focusing on stationary equilibria and assuming the existence of a representative consumer,[5] Appendix 1 shows that the consumption ratio $\delta$ is given by

$$\sigma\delta^{\sigma-1} = \frac{\delta^{\sigma}}{\rho - \Gamma(\gamma^{\sigma}-1)}\left[\frac{\alpha^2 N + (1-\alpha^2)L}{LN}(\gamma^{\sigma}-1)\lambda\frac{L}{\alpha} + \Gamma\sigma\gamma^{\sigma}\frac{L}{n}(1-N^{-1})\frac{1-\alpha}{\alpha}\right]. \tag{17}$$

This equation can be simplified in an obvious way but as it stands it better reveals its economic meaning. It restates the first order condition (15) and says that the consumption ratio $\delta$ is chosen such that marginal utility from current consumption on the left hand side is equal to discounted expected values of the gain from a new technology times the marginal arrival rate (the first term in brackets) plus the arrival rate times the marginal value from investment (the second term in brackets). Expectations manifest themselves in the discount factor. The expected value of a constant income stream that ends at some Poisson distributed point in time is given by income per period divided by the time preference rate plus the arrival rate: The risk that the income stream stops increases the discount factor. Here, discounting takes place at the time preference rate plus the arrival rate *minus* the arrival rate times the increase in utility $\gamma^{\sigma}$ from a new technology, $\rho - \Gamma(\gamma^{\sigma}-1)$. The discount rate is lower than the time preference rate since each individual gains from the new technology.

By having derived individual consumption, aggregate consumption and aggregate investment are determined as well and the equilibrium of the model is completely described. Aggregate consumption equals the number of households $N$ times individual consumption and aggregate investment is given by

$$I = (1-\delta)(w+\pi)N. \tag{18}$$

Using this aggregate investment equation together with the financing equation (8) allows to express labour demand in the R&D sector as a function of the individual consumption ratio $\delta$. Inserting this demand function into the labour market clearing condition allows to express employment $L$ in the intermediate good sector $X$ as a function of $\delta$. Solving for $\delta$ gives,

$$\delta = \frac{L}{\alpha^2 N + (1-\alpha^2)L}. \tag{19}$$

Equations (17) and (19) jointly determine the individual saving ratio $\delta$ and aggregate employment in production $L$ which is identical to saying aggregate investment.

[5] Assuming a representative consumer especially implies that all households own the same share $s = N^{-1}$ in the current monopolist. Stationary equilibria are equilibria where factor allocation is identical for all technologies.

## 2. Strategic Complementarities in R&D Finance

The individual consumption ratio $\delta$ in (17) is a function of aggregate variables (and of constants). Most importantly, individual consumption and therefore investment decisions depend on aggregate consumption. This introduces a strategic complementarity into individual investment decisions and distorts the decentralised equilibrium. Aggregate variables affect individual consumption decisions via the arrival rate and via the sharing rule $s = iI^{-1}$ implicit in (13). Higher aggregate investment increases the arrival rate and, at the same time, decreases an individual's share in the monopolist's profits. The dependence of the probability of payoff of an individual's investment on aggregate investment via the arrival rate does by itself *not* distort individual investment decisions. This impact would be taken into consideration by a planner as well and is also present in models with Pareto optimal decentralised equilibria of the type presented here (Wälde, 1998, c 7). It is the sharing rule which causes the strategic complementarity.

In a context as considered here, this sharing rule is natural for two reasons. First, when the R&D sector is modelled as essentially one large R&D project, all individuals must invest in this single project. There must therefore be some rule that determines how returns from R&D are shared and a straightforward rule is the one used here. But even if the R&D sector had been modelled to consist of many distinguishable R&D projects, individuals would find it profitable for reasons of risk-diversification to allocate some of their savings to *each* R&D project. Each single R&D project would therefore be funded by many individuals and, again, a sharing rule would be required. Second, R&D, as conceived here, is an investment form that leads to a payoff which is independent of total investment. The payoff of R&D is a new technology and this new technology has a certain fixed value. An increase in total investment only increases the arrival rate. Again, a sharing rule is required to distribute the fixed payoff among investors. If capital were accumulated, the amount of capital available next period would depend on the amount of savings this period. No sharing rule would then be required and no complementarities would arise.

Equilibrium in this model is achieved when the individual consumption curve (17) and the aggregate consumption curve (19) are both satisfied which holds at their intersection point illustrated in Fig. 1. The aggregate consumption curve starts in the origin and rises in $L$. The individual consumption curve intersects the $\delta$ axis at some positive level[6] and decreases if

$$\lambda N(\gamma^{\sigma} - 1) < \rho - \Psi^{-1}\alpha^2\lambda N(\gamma^{\sigma} - 1)^2, \tag{20}$$

where $\Psi = (1-\alpha^2)(\gamma^{\sigma} - 1) + \sigma\gamma^{\sigma}(N-1)(1-\alpha)$ is a collection of constants. It must hold that $\lambda N(\gamma^{\sigma} - 1) < \rho$ since otherwise, the integral (10) would not converge. This can be seen from (17) or more clearly later from (21) which

[6] The model is set up under the assumption that employment in the consumption good sector $L$ is strictly positive. The individual consumption curve only holds for $L > 0$ and there is no economic interpretation for $\delta(L = 0)$.

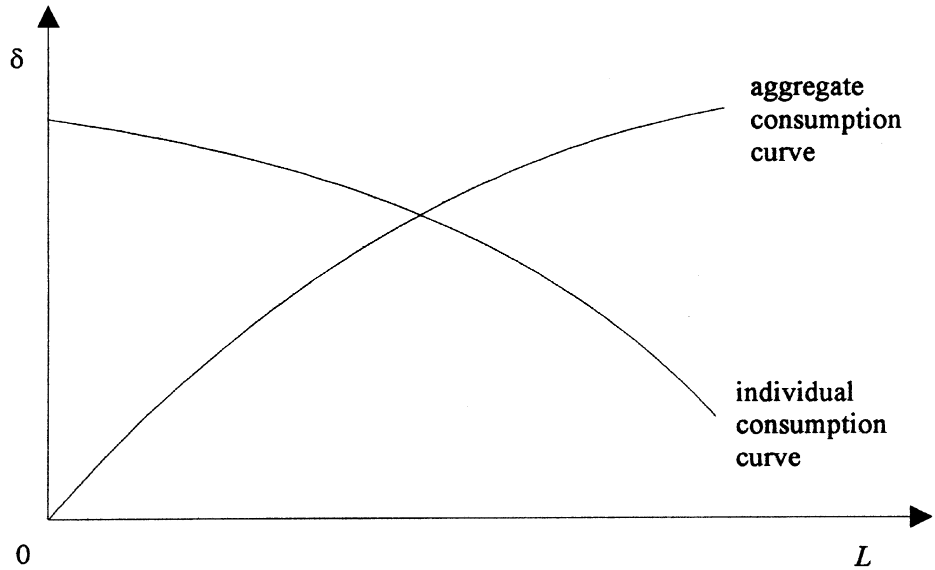


Fig. 1. *The equilibrium consumption level*

gives the value of the optimal program. Hence, if the expected growth rate is not too high, this condition holds.[7]

Slightly rewriting (17) allows us to derive comparative static properties of the equilibrium growth rate by studying how curves in Fig. 1 shift. It is straightforward to see that an increase in $\gamma$, the increase in total factor productivity after an innovation, decreases the consumption ratio $\delta$ and shifts the individual consumption curve in Fig. 1 to the left. The amount of labour in the R&D sector therefore rises and the expected growth rate rises. An increase in the marginal arrival rate $\lambda$ increases the consumption ratio and decreases the amount of labour allocated to R&D. The effect on the expected growth rate is therefore ambiguous. A lower time preference rate unambiguously boosts growth. These results confirm the findings of Aghion and Howitt.

The first central difference resulting from an explicit study of consumer behaviour is the strategic complementarity between aggregate investment and individual investment. To see why this complementarity does not appear in Aghion and Howitt's model and in order to see how these two models are linked to each other, consider the value a household attaches to holding a share $s$ in the current monopolist, $V(s)$. Since from (37) in Appendix 1, $V(s,\, t+1) = \gamma^{\sigma} V(s,\, t)$, this value follows from the Bellman equation (14) and reads

$$V(s) = \frac{u(c)}{\rho - \Gamma(\gamma^{\sigma} - 1)}. \qquad (21)$$

[7] If households neglected the effect of their consumption decision on the arrival rate, the second term on the right-hand side of (20) would vanish and the condition would hold.

The value from holding a share $s$ in the current monopolist equals utility derived from a consumption stream where consumption jumps by $\gamma$ at an arrival rate $\Gamma$. In Aghion and Howitt's setup, the value a firm attaches to holding a monopoly is assumed to be given by (their equation 2.12)

$$V = \frac{\pi_x}{\rho + \Gamma}.$$

The value of being a monopolist equals profit flows $\pi$ discounted at the time preference rate plus the arrival rate.

Comparing these two expression reveals three differences. At the most obvious level, the present paper allows for risk aversion. Values are therefore expressed in utility units. If we assumed risk neutrality ($\sigma = 1$), expression (21) would read

$$V(s) = \frac{\delta(w + N^{-1}\pi_y + s\pi_x)}{\rho - \Gamma(\gamma - 1)}.$$

This shows the second difference. Current income of households is given by profit flows from the current monopolist plus profit payments from the consumption good sector and wage income. In order to make models more similar, one would have to assume that the economy consists of entrepreneurs whose income is given by profits of the monopolist only and households whose income is given by labour and profit income from the consumption good sector.

Finally, in the Aghion and Howitt framework, a monopoly is owned by exactly one investor ($s = 1$). The current owner of the monopoly therefore has no incentive to invest in R&D, her consumption ratio $\delta$ equals unity. In a representative household framework as used here, each household owns a fraction of the current monopolist and therefore has an incentive to invest in the next technology. This explains why individuals discount here at a rate lower than the time preference rate (since gains from a new technology are positive) while a monopolist in Aghion and Howitt's framework discounts at a rate higher than the time preference rate (since the current monopolist loses from a new technology). Further, when a monopoly is to be owned by one investor only, each entrepreneur must run her own R&D project. Then, no sharing of monopoly profits takes place and the complementarity identified here is absent.

In short, necessary conditions for obtaining Aghion and Howitt's result are risk neutrality, a division of agents into entrepreneurs and households, that only entrepreneurs are allowed to save and that each entrepreneur invests in exactly one R&D project.

## 3. Welfare Analysis

We have seen in the last section that the sharing rule introduces an externality. It remains to be seen whether aggregate and individual investment are strategic complements or strategic substitutes in the sense of Cooper and John (1988). Is investment in the decentralised economy too high or too low?

### 3.1. *The Social Planner's Solution*

We let a planner choose aggregate consumption $y$ in order to maximise a social welfare function (10) with instantaneous utility given by $N$ times the instantaneous utility function (11) subject to a 'resource constraint' that reads

$$dt = dq.$$

In contrast to individuals who trade off current consumption to the arrival rate and expected future shares in monopolists, the planner trades off current consumption to the arrival rate of the R&D process only. Further, the asset valued by the planner is not an individual's share $s$ but the current state of the technology $t$. The Bellman equation therefore reads

$$\rho V(t) = \max_{y} \left\{ Nu\left(\frac{y}{N}\right) + \Gamma[V(t+1) - V(t)] \right\},$$

where the arrival rate is $\Gamma = \lambda[N - (y\gamma^{-t})^{1/\alpha}]$ which follows from inserting the factor market clearing condition (9) and the production function (1). The first order condition reads

$$u'\left(\frac{y}{N}\right) + [V(t+1) - V(t)]\frac{d\Gamma}{dy} = 0. \tag{22}$$

Marginal utility from consumption plus the increase in the value of the optimal plan from the next technology times the marginal arrival rate must be zero.

Following the same steps as above and assuming a value function of the form $V(t) = \vartheta\gamma^{t\sigma}$ shows that the optimal number of researchers chosen by the planner is given by

$$n = \frac{1}{1-\alpha\sigma}\left(N - \frac{\alpha\sigma}{\gamma^{\sigma}-1}\frac{\rho}{\lambda}\right). \tag{23}$$

This is a risk-adjusted optimal number of researchers. If households were risk neutral ($\sigma = 1$) the optimal number would equal the optimal number derived by Aghion and Howitt, as can be seen by solving their equation (4.5) for $n^*$. The effect of parameter changes is the expected one for most parameters. The socially optimal number of researchers increases in the active labour force $N$, in the increase in total factor productivity $\gamma$, in the output elasticity $\alpha$, in the marginal arrival rate $\lambda$ and it increases as agents become more patient.

The effect of risk aversion as captured by $\sigma$, however, is ambiguous.[8] This can most easily be understood by considering the planner's optimality condition (22) which can be expressed as

$$\sigma\frac{\alpha}{\lambda} = (N - n)\frac{\gamma^{\sigma}-1}{\rho - \lambda n(\gamma^{\sigma}-1)}. \tag{24}$$

[8] The sign of the derivative of $n$ with respect to $\sigma$ has to take parameter restrictions into consideration. The first restriction requires that $n > 0$ while the boundedness condition requires $\rho > \lambda N(\gamma^{\sigma} - 1)$. Even taking these restrictions into account shows that the derivative can have either sign.

An increase in $\sigma$ increases both marginal utility represented by the left-hand side and discounted gains from future technologies represented by the right-hand side. In principle, either effect can dominate. It is clear from (23), however, that the number of researchers increases as households become less risk averse when $N$ is sufficiently large while it falls when $N$ is sufficiently small.

### 3.2. *Two Market Failures*

If one wants to compare the number of researchers under central and decentral factor allocation, it is useful to compare first order conditions. By explicitly computing the value functions, the planner's first order condition (22) can be rewritten as

$$u'(c) + \frac{y^{\sigma}(\gamma^{\sigma}-1)}{\rho - \Gamma(\gamma^{\sigma}-1)} \frac{d\Gamma}{dy} = 0. \tag{25}$$

The difference in the values of technology $t+1$ and technology $t$ therefore equals the value of the optimal program (compare (21) which gives the value of the optimal program of an individual) times the difference in instantaneous utility between the current and the next utility, $\gamma^{\sigma}-1$. Computing the difference in the values of technology $t+1$ and technology $t$ for the decentralised economy allows us to rewrite an individual's first order condition (15) by using (19) and (32), (36) and (37) from Appendix 1 as

$$u'(c) + \frac{c^{\sigma}(\gamma^{\sigma}-1)}{\rho - \Gamma(\gamma^{\sigma}-1)} \frac{d\Gamma}{dc} + \Gamma \frac{d}{dc} V(iI^{-1},\ t+1) = 0. \tag{26}$$

There are two differences between these first order conditions. First, there is an additional term on the right-hand side in the individual's first order condition (26) which reflects division of shares in the next monopolist. This is unambiguously negative, and since the second derivative of the planner's problem must be negative (in order to obtain a maximum), by this effect, total consumption $y$ in a decentralised economy tends to be lower (cf. Appendix 2 for a proof). Second, individuals take into consideration the effect an improved technology has on own consumption $c$ in contrast to the central planner who focuses on aggregate consumption $y$. This has an increasing effect on aggregate consumption in a decentralised economy. The overall effect is therefore ambiguous[9] and no prediction can be made whether the decentralised growth rate is higher or lower than the central planner's growth rate.

Aghion and Howitt stress four differences between the centralised and the decentralised equilibrium. The difference in the discount factor and their 'business stealing effect' vanish in a representative consumer approach. When

[9] The derivative of the arrival rate with respect to individual and aggregate consumption in these two first order conditions is identical as an individual values an increase in the arrival rate due to her own consumption in exactly the same way as a planner does (cf. (34)).

all households own shares in n the current monopolist and finance R&D for the next monopolist, they internalise the 'business stealing effect' and they discount with a discount rate that internalises the gain from the new technology.

The distortion caused by the monopolist manifests itself in the Aghion-Howitt approach since R&D firms take wages as given. When households optimally choose consumption, they take wages as given as well (cf. Appendix 1). The distortion between the perfectly-competitive R&D sector and the monopolistically influenced consumption good sector does not distort the households investment decision, however, since households explicitly take into consideration the effect of their consumption choice on the arrival rate (cf. (34)). If households bought consumption and investment goods taking prices of these goods as given, the goods market distortion would translate into an investment distortion. Internalising the effect of their investment on the arrival rate, households look behind this 'price veil' and the goods market distortion does not lead to an investment distortion. Their final difference is the 'appropriability effect'. This effect appears here as well and is the second one identified above.[10]

The second central difference, in addition to the market failure arising from the sharing rule, obtained from analysing explicitly household behaviour therefore is that three out of four market failures identified by Aghion and Howitt vanish in a representative agent framework.

## 4. Conclusion

This paper extended the creative destruction paper by Aghion and Howitt (1992) by studying household maximisation behaviour in more detail. Households are risk averse and determine by their consumption choice the amount of resources allocated to R&D. Since R&D is governed by Poisson uncertainty, solving the household maximisation problem is non-standard and interesting in its own right.

While comparative static results obtained by Aghion and Howitt continue to hold, joint financing of R&D projects by many households has been shown to introduce a strategic complementarity via the dependence of individual investment decisions on aggregate investment. This market failure stems from the sharing rule which determines how gains from successful R&D are distributed among investors. This rule is natural in models that combine representative agent maximisation and uncertain R&D. A representative agent analysis implies that all individuals invest in R&D and therefore some sharing rule must be used and uncertain R&D implies that the payoff from a successful R&D project – in contrast to capital accumulation – does not depend on total

[10] Note that this is not obvious. When decentralising an economy, this does not necessarily imply an 'appropriability' effect. When an individual optimally decides on how much to save, she takes the interest rate as given and takes into account only the effect of her savings on own capital income (and not for examples on her own wage or the wage rate of others). Nevertheless, there is no appropriability effect there.

investment. This complementarity was not present in the Aghion and Howitt paper.

Turning to a welfare comparison of the centralised and decentralised economy, it has been shown that the sharing rule induces individuals to invest too much. The second difference between the planner and the decentralised equilibrium is the appropriability effect which tends to reduce aggregate investment. This effect is similar to one of the four market failures identified by Aghion and Howitt. Their remaining three market failures disappear, however, when a representative agent approach is chosen.

*University of Dortmund and The World Bank*

## References

Aghion, P. and Howitt, P. (1992). 'A model of growth through creative destruction.' *Econometrica*, vol. 60, pp. 323–51.
Bismut, J. M. (1975). 'Growth and intertemporal allocation of risks.' *Journal of Economic Theory*, vol. 10, pp. 239–57.
Bourguignon, F. (1974). 'A particular class of continuous-time stochastic growth models.' *Journal of Economic Theory*, vol. 9, pp. 141–58.
Chow, G. C. (1979). 'Optimum control of stochastic differential equation systems.' *Journal of Economic Dynamics and Control*, vol. 1, pp. 143–75.
Cooper, R. and John, A. (1988). 'Coordinating coordination failures in Keynesian models.' *Quarterly Journal of Economics*, vol. 103, pp. 441–63.
Dixit, A. K. and Pindyck, R. S. (1994). *Investment Under Uncertainty*. Princeton: Princeton University Press.
Duffie, D. and Epstein, L. G. (1992). 'Stochastic differential utility' (Appendix C with Costis Skiadas). *Econometrica*, vol. 60, pp. 353–94.
Eaton, J. (1981). 'Fiscal policy, inflation and the accumulation of risky capital.' *Review of Economic Studies*, vol. 48, pp. 435–45.
Merton, R. C. (1969). 'Lifetime portfolio selection under uncertainty: the continuous-time case.' *Review of Economics and Statistics*, vol. 51, pp. 247–57.
Merton, R. C. (1971). 'Optimum consumption and portfolio rules in a continuous-time model.' *Journal of Economic Theory*, vol. 3, pp. 373–413.
Merton, R. C. (1975). 'An asymptotic theory of growth under uncertainty.' *Review of Economic Studies*, vol. 42, pp. 375–93.
Merton, R. C. (1990). *Continuous-Time Finance*. Oxford: Basil Blackwell.
Obstfeld, M. (1994). 'Risk-taking, global diversification, and growth.' *American Economic Review*, vol. 84, pp. 1310–29.
Ross, S. M. (1993). *Introduction to Probability Models*. 5th edition. London: Academic Press.
Turnovsky, S. J. (1993). 'Macroeconomic policies, growth, and welfare in a stochastic economy.' *International Economic Review*, vol. 34, pp. 953–81.
Turnovsky, S. J. (1995). *Methods of Macroeconomic Dynamics*. Cambridge, MA: MIT Press.
Wälde, K. (1998). *Poisson-Ramsey Economies: On the Endogeneity of Cycles in Economic Growth*. Dortmund: Manuscript (Habilitationsschrift).

## Appendix 1

Given the Bellman equation (14) and the first order condition (15), we guess that optimal consumption is given by (16),

$$c_t = \delta_t[w_t + \pi_t(s_t)],$$

where $\delta_t$ is independent of household specific variables (wage and capital income or investment) and that value functions read

$$V(s_t,\ t) = \vartheta[w_t + \pi_t(s_t)]^\sigma,\ V(i_t I_t^{-1},\ t+1) = \vartheta[w_{t+1} + \pi_{t+1}(i_t I_t^{-1})]^\sigma,$$

where $\vartheta$ is a constant. Variables that depend on the technology $t$ are indexed by $t$ in this appendix. Note that the share in the next monopolist $t+1$ is determined by investment under technology $t$, $s_{t+1} = i_t I_t^{-1}$. The remainder of this appendix shows that these guesses indeed provide a solution and computes the share $\delta_t$.

Let us start by computing value functions. By (5), (2), (1) and (4), wages are given by

$$w_t = \alpha p_t = \alpha^2 \gamma^t x_t^{\alpha-1} = \alpha^2 y_t x_t^{-1} = \alpha^2 y_t L_t^{-1}. \tag{27}$$

By (6), (3) and (1), and by assuming that each household receives the same share $N^{-1}$ of profits of firms in the *consumption* good sector, capital income $\pi$ is given by

$$\pi_t(s_t) = N^{-1}\pi_{y_t} + s_t \pi_{x_t} = (N^{-1} + \alpha s_t)\pi_{y_t} = (N^{-1} + \alpha s_t)(1-\alpha) y_t. \tag{28}$$

Current income therefore amounts to

$$w_t + \pi_t(s_t) = \left[\frac{\alpha^2}{L_t} + \left(\frac{1}{N} + \alpha s_t\right)(1-\alpha)\right] y_t. \tag{29}$$

and value functions read

$$V(s_t,\ t) = \vartheta\left\{\left[\frac{\alpha^2}{L_t} + \left(\frac{1}{N} + \alpha s_t\right)(1-\alpha)\right] y_t\right\}^{\sigma} \tag{30}$$

and

$$V\left(\frac{i_t}{I_t},\ t+1\right) = \vartheta\left\{\left[\frac{\alpha^2}{L_{t+1}} + \left(\frac{1}{N} + \alpha\frac{i_t}{I_t}\right)(1-\alpha)\right] y_{t+1}\right\}^{\sigma}. \tag{31}$$

Since individuals take wage and capital income as independent of own consumption, the value of holding a share $s_t$ is independent of her consumption choice,

$$\frac{d}{dc_t} V(s_t,\ t) = 0. \tag{32}$$

Current consumption does affect future share holdings, however, and we have

$$\frac{d}{dc_t} V\left(\frac{i_t}{I_t},\ t+1\right) = \vartheta\sigma\left\{\left[\frac{\alpha^2}{L_{t+1}} + \left(\frac{1}{N} + \alpha\frac{i_t}{I_t}\right)(1-\alpha)\right] y_{t+1}\right\}^{\sigma-1} \frac{d(i_t/I_t)}{dc_t}\alpha(1-\alpha) y_{t+1} \tag{33}$$

where

$$\frac{d(i_t/I_t)}{dc_t} = \frac{d[w_t + \pi_t - c_t/(\overline{I}_t + w_t + \pi_t - c_t)]}{dc_t} = \frac{i_t - I_t}{I_t^2}.$$

This formulation captures the idea that an individual takes the effect of own consumption on own and aggregate investment into consideration but considers aggregate investment net of own investment, $\overline{I}_t$, as given. The derivative of the value of investing $i_t$ is given by the marginal value of income next period which is the first term, $\vartheta\sigma(\cdot)^{\sigma-1}$, times the decrease in sector $X$ capital income *after* an innovation from more consumption today, $d\pi_{x_{t+1}}/dc_t$, which is the second term. Further, the derivative of the arrival rate with respect to own consumption is given by

$$\frac{d}{dc_t}\Gamma = \lambda\frac{d}{dc_t}\left[N - \left(\frac{\overline{c}_t + c_t}{\gamma^t}\right)^{1/\alpha}\right] = -\lambda\frac{1}{\alpha}\left(\frac{y_t}{\gamma^t}\right)^{(1-\alpha)/\alpha}\gamma^{-t} = -\lambda\frac{1}{\alpha}L_t^{1-\alpha}\gamma^{-t} = -\lambda\frac{L_t}{\alpha}y_t^{-1}. \tag{34}$$

Again, this formulation lets individuals take consumption by others $\overline{c}_t$ as given and compute only the effect of own consumption on the arrival rate. The expression for employment in the R&D sector, $n_t = N - (y_t \gamma^{-t})^{1/\alpha}$, follows from the labour market clearing condition (9) and the production function (1).

After having computed derivatives for the first order condition in all generality, we can now impose our assumption of a representative consumer. When all consumers are identical,

$$iI^{-1} = s = N^{-1}, \tag{35}$$

current income (29) is given by

$$w_t + \pi_t(N^{-1}) = \left(\frac{\alpha^2}{L_t} + \frac{1-\alpha^2}{N}\right) y_t = \frac{\alpha^2 N + (1-\alpha^2) L_t}{L_t N} y_t \equiv \Phi y_t. \tag{36}$$

We further restrict attention to stationary equilibria where the allocation of labour is invariant across technologies, $L_t = L$. As a consequence, $\delta$ is identical across technologies as well. This rules out cycles by assumption and allows to write the ratio $\Phi$ without time index. The value functions (30) and (31) are then given by

$$V(N^{-1}, t) = \vartheta(\Phi y_t)^\sigma, \quad V(N^{-1}, t+1) = \vartheta(\Phi y_{t+1})^\sigma \tag{37}$$

and the derivative (33) reads

$$\begin{aligned} \frac{d}{dc_t} V(N^{-1}, t+1) &= -\vartheta\sigma(\Phi y_{t+1})^{\sigma-1} \frac{I_t - i_t}{I_t^2} \alpha(1-\alpha) y_{t+1} \\ &= -\vartheta\sigma(\Phi y_{t+1})^{\sigma-1} \frac{L}{n} (1 - N^{-1}) \frac{1-\alpha}{\alpha} \gamma. \end{aligned}$$

where the last equality follows from

$$\frac{I_t - i_t}{I_t^2} = \frac{L_t}{\alpha^2 y_t n} (1 - N^{-1})$$

using (8), (27) and (35). Finally, consumption (16) is given by

$$c_t = \delta \Phi y_t.$$

Inserting this all into the consumption first order condition (15) gives

$$\sigma(\delta\Phi y_t)^{\sigma-1} = \vartheta(\Phi y_t)\sigma(\gamma^{\sigma-1})\lambda \frac{L}{\alpha} y_t^{-1} + \Gamma\vartheta\sigma(\Phi y_{t+1})^{\sigma-1} \frac{L}{n} (1 - N^{-1}) \frac{1-\alpha}{\alpha} \gamma.$$

Dividing by current income (to the power of $\sigma - 1$) and some rearrangement simplifies this to

$$\sigma\delta^{\sigma-1} = \vartheta\left[\Phi(\gamma^{\sigma-1})\lambda \frac{L}{\alpha} + \Gamma\sigma\gamma^\sigma \frac{L}{n} (1 - N^{-1}) \frac{1-\alpha}{\alpha}\right]. \tag{38}$$

Let us now employ the Bellman equation to eliminate $\vartheta$ from this first order condition. The Bellman equation (14) reads, with optimal investment and the arrival rate inserted,

$$\begin{aligned} \rho\vartheta(\Phi y_t)^\sigma = (\delta\Phi y_t)^\sigma + \Gamma\vartheta(\Phi y_t)^\sigma(\gamma^\sigma - 1) &\Leftrightarrow \rho\vartheta = \delta^\sigma + \Gamma\vartheta(\gamma^\sigma - 1) \\ &\Leftrightarrow \vartheta = \frac{\delta^\sigma}{\rho - \Gamma(\gamma^\sigma - 1)}. \end{aligned} \tag{39}$$

Inserting into the first order condition (38) gives expression (17) for $\delta$ in the text.

## Appendix 2

The planner's first order condition (25) is of the form $f(y_{\text{plan}}) = 0$, where $y_{\text{plan}}$ is planned aggregate consumption. The first order condition of a household then reads $f(y_{\text{dec}}) = -N^{\sigma}\Gamma(d/dc)V(iI^{-1}, t+1) > 0$, where $y_{\text{dec}}$ is decentralised aggregate consumption. The second derivative of the planner's maximisation problem must be negative, otherwise $y_{\text{plan}}$ would not constitute a maximum, $f'(y_{\text{plan}}) < 0$. When $f'(y) < 0$ for all aggregate consumption levels $y$ or at least for the relevant range considered here, $f(y_{\text{plan}}) = 0$ and $f(y_{\text{dec}}) > 0$ imply $y_{\text{dec}} < y_{\text{plan}}$.

An appendix with further derivations is available upon request.