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Proof of Global Stability, Transitional Dynamics, and International Capital Flows in a Two-country Model of Innovation and Growth

Klaus Wälde

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Global stability properties of dynamic two-country models can be easily studied in the case of perfect international capital flows. With internationally constant relative productivities, balanced-growth path values for factor prices will hold on any path leading to the balanced-growth path unless one country experiences a period of no innovation. Innovation rates converge in the case of perfect international knowledge spillovers but long-run consumption levels and trade patterns are path-dependent. GDP per capita is predicted to converge slowly despite the presence of perfect international capital markets and no explicit inclusion of adjustment costs. The trade balance of the rich country is initially positive but after some time turns into a deficit.

Keywords: convergence, international trade, international capital flows, stability, endogenous growth.

JEL classification: F12, O41.

1 Introduction

Issues of international trade are usually analyzed in a long-run equilibrium framework where all trading economies are in a steady state and factor allocations are constant. This is especially true for investigations into dynamic effects of international trade. The usual assumption is that economies are on a balanced-growth path (BGP) and all findings on wages, trade patterns, consumption levels, or growth rates are then restricted to this possible state of the world. The aim of this paper is to describe the evolution of two trading economies from the moment they begin to trade until they reach their common BGP. We therefore do not restrict our attention to two disconnected points (autarky and trade equilibrium in a static analysis) or periods in time (autarky and trade BGP in a dynamic model) but trace the behavior of variables at *every* point in time from autarky to a long-run trade equilibrium. Such an approach has the advantage to predict the effect of trade on factor prices and consumption for the moment countries open up to trade and not only for the long run, to provide exact conditions for convergence and thereby to suggest possible policy measures to foster a catching-up. It further allows to determine certain properties of the BGP, such as consumption levels or trade balances, that cannot always be determined by considering the BGP alone.

Since such an approach requires a global-stability analysis of the model under consideration, this paper adds to a recent series of contributions on transitional dynamics in endogenous-growth models. The general motivation underlying those papers is to complement an analysis of the BGP itself by a study of its stability properties since a priori it is not clear whether trajectories leading to a BGP (equilibrium paths in the following, a precise definition follows in Sect. 3) exist for all possible, historically given, initial conditions. Once this has been established, the task consists in determining properties of those equilibrium paths given that a real world economy is always much more likely to be in transition towards a BGP rather than on the BGP itself. Mulligan (1991) and Mulligan and Sala-i-Martin (1993) were among the first to analyze transitional dynamics in endogenous-growth models. The basic idea is to consider *ratios* rather than levels of variables[1] which is applied by Mulligan and Sala-i-Martin (1993) to a class of models characterized by the accumulation of two factors of production. One of the models they consider is the Lucas (1988) model for which a numerical analysis of global-stability behavior is given. They find that for the parameter values under consideration, the long-run equilibrium is globally saddle-path stable, i.e., all variables, independently of initial conditions for state variables, converge monotonically to their BGP trajectories. An analytical study of global-stability behavior provided by Caballé and Santos (1993) confirms the global-stability results and provides additional outside-BGP findings on the *levels* of capital and human capital. A thorough analytical investigation of the local-stability properties by Benhabib and Perli (1994) emphasizes, however, that there are parameter values (apparently not considered by Mulligan and Sala-i-Martin, 1993) for which the BGP of the Lucas (1988) model is not saddle-path stable, but can rather be reached by a continuum of paths.[2]

Clearly, every analysis of dynamics of a model faces a trade-off

1 An approach previously used, e.g., in resource economics by Suzuki (1976).

2 Segerstrom (1994) analyzes the stability properties of the quality-ladders model by Grossman and Helpman (1991c).

between tractability and the complexity of the model. This paper definitely opts for strong results and therefore takes as a framework the simplest possible model which still has all ingredients of a typical two-country world of innovation and growth. The model, which is presented in Sect. 2, is a Grossman and Helpman (1991) type two-country world model with perfect international capital flows. Firms are therefore not restricted to domestic savings for financing their R&D expenditures, a fact that will turn out to be crucial for the analysis.

The first analysis of out-of-BGP behavior of a trade model of that kind was undertaken by Grossman and Helpman (1990) themselves, in their first paper on the impact of trade on growth. They find that an equilibrium path towards a BGP is characterized by constant relative wages and prices and that the BGP is saddle-path stable. As they mention themselves these findings are subject to the condition that innovation rates in both countries are positive during the transition period and therefore conclude that their findings hold in the vicinity of a BGP only. They do not explicitly discuss the method allowing a stability analysis, however. Neither are further properties and implications of the transition period nor of the condition of positive innovation rates provided.

Devereux and Lapham (1994) study the stability properties of the Rivera-Batiz and Romer (1991) model on economic integration and endogenous growth, whose analysis took only the model's BGP into consideration. Devereux and Lapham focus on the knowledge-driven variant of Rivera-Batiz and Romer's model, which basically is a two-country version of Romer (1990). This implies that an important assumption has to be made about international spillovers of knowledge, as pointed out by Grossman and Helpman (1991a, chap. 8). They show that knowledge spillovers which are restricted to the country where knowledge resulted as a by-product from the R&D activity imply a local instability of the BGP, i.e., innovation rates of countries will never converge. Devereux and Lapham (1994) make a similar point: since the Rivera-Batiz and Romer model has national spillovers only, it can be shown that a BGP is a knife-edge result which holds only if knowledge stocks (i.e., number of firms active in the market) are identical prior to a trade liberalization. If this condition fails to hold, a country that has a momentary advance in terms of domestic knowledge over its trading partners will end up being the only country doing R&D. Thus the BGP with equal innovation rates in both countries is unstable. What the analysis does not provide, however, is an analysis of the model properties in the case of international knowledge spillovers where global convergence can be expected as the following analysis indicates. This is quite understandable, however, given the complexity of the model.

Other papers have studied transitional aspects in trade models, though with a different emphasis and different underlying questions. Backus et al. (1992) study international transmission effects of technology shocks in a two-country RBC model. Though not their emphasis, this could be interpreted to represent a trade liberalization between two countries that use different technologies. Their framework is similar to Kydland and Prescott (1982) and therefore does not allow to draw conclusions about transitional characteristics of the model under consideration here. The same is true for Ono and Shibata (1992) who study the effects of an unilateral increase of productivity in a two-country model with physical capital accumulation and perfect foresight.

The main contributions of the present paper are as follows. Section 3 presents the basic mechanism and economic background why a simple proof of global-stability analysis in this and related models is possible and presents the proof itself. It will turn out that stability can be easily proven as long as both countries innovate during the transition period.

Section 4 shows why wages and prices must be constant on any path leading to the BGP and, equivalently, why (the failure of) long-run factor-price equalization directly translates to the entire equilibrium path. The section proceeds by highlighting the precise mechanism leading to a catching-up of the backward country and the implications for innovation rates. One finding will be that the growth rate of the world as a whole is constant during the entire adjustment period. Due to this result, the condition required for the proof of global stability, that both countries innovate while trading, can be studied in detail. Circumstances under which it holds, hence under which the proof of global stability is valid, are given. A discussion of the effects if one country stops innovating after opening up to trade follows. The section then focuses on the implications of international capital flows on expenditure levels, GDP convergence, and the trade balance. It will be shown that long-run expenditure levels are path-dependent, a fact sometimes neglected in the analysis of growth models with international capital flows. The findings on convergence of GDP during the transition period are in stark contrast to, but more realistic than the prediction of a neoclassical two-country model with international capital flows. A reversal of the trade balance from a deficit to a surplus is shown to be a feature of the catching-up process. The section closes by deriving a precise measure of the speed of GDP convergence and by illustrating the behavior of the world economy in a figure containing the BGP, equilibrium paths, and the saddle plane.

A natural question to ask is how robust these findings are to changes. Hence, Sect. 5 discusses which results carry over to other models and what type of models can be analyzed in the same way as presented here. The focus is on extended versions of the present model, countries with different technologies or preferences and imperfect international knowledge spillovers. Finally, conclusions are drawn.

2 A Typical Two-country World of Innovation and Growth

The model consists of two countries producing one differentiated final good using homogeneous internationally immobile labor as input. Countries are assumed to be at or after a certain point in time denoted by t_{trade} when they start to interact actively by trading final goods and financial capital, which will give rise to intra-industrial trade and interest-rate equalization, and passively by experiencing perfect international knowledge spillovers. Since models of this type are extensively discussed in Grossman and Helpman (1991a, chap. 9.1), the description will be short.

Utility of a consumer living in country i ($i = \text{A}, \text{B}$) at time t stems from a stream of future consumption, discounted at the time preference rate ρ, and is given by $U^i(t) = \int_t^\infty \exp[-\rho(\tau - t)]u^i(\tau)\,\mathrm{d}\tau$, where instantaneous utility $u^i(\tau)$ depends on consumption of varieties k of a differentiated final good x produced in both countries. Letting n^i indicate the "number" of varieties currently produced in country i, instantaneous utility amounts to $u^i(\tau) = \log\left(\int_0^{n^\text{A}+n^\text{B}} (x^k)^\alpha\,\mathrm{d}k\right)^{1/\alpha}$, $0 < \alpha < 1$, where the constant elasticity of substitution between any two varieties is given by $\varepsilon = 1/(1-\alpha)$. These preferences imply that demand of country i as a whole for a variety k is a function of prices p^k, of the number of differentiated goods, and of current expenditure E^i, $x^k = (p^k)^{-\varepsilon} E^i \left(\int_0^{n^\text{A}+n^\text{B}} (p^{k'})^{1-\varepsilon}\,\mathrm{d}k'\right)^{-1}$, $k \in [0, n^\text{A} + n^\text{B}]$. Optimal allocation of expenditure over time is then derived by reinserting this demand function into the intertemporal utility function and maximizing it with respect to expenditure subject to an intertemporal budget constraint. Aggregating over consumers, the consumption side of economy i as a whole is then given by an equation determining allocation of expenditure,

$$\frac{\dot{E}^i(t)}{E^i(t)} = r(t) - \rho \ , \tag{1}$$

where $r(t)$ is the world-wide interest rate; a budget constraint for each country of the form

$$E^i(t) + \dot{A}^i(t) = r(t)A^i(t) + w^i(t)L^i \ , \tag{2}$$

which equates current expenditure and change of the value of asset holdings $\dot{A}^i(t)$ of households of that country to asset and labor income, $r(t)A^i(t)$ and $w^i(t)L^i$, respectively; two transversality conditions, $\lim_{\tau\to\infty} \exp[-\rho\tau]A^i(\tau)/E^i(\tau) = 0$; and aggregate world demand for a variety produced in country i, given by

$$x^i = \frac{(p^i)^{-\varepsilon}}{n^{\mathrm{A}}(p^{\mathrm{A}})^{1-\varepsilon} + n^{\mathrm{B}}(p^{\mathrm{B}})^{1-\varepsilon}}(E^{\mathrm{A}} + E^{\mathrm{B}}) \; . \tag{3}$$

In expressing the last equation, the fact has been used that, as will turn out when looking at the production side, all varieties in country i are equally priced.

The production side is characterized by two activities: production of varieties for which blueprints have already been developed and development of new blueprints. The production process takes place under constant returns to scale and follows the simplest production function possible, $x^k = L_x^k$, where L_x^k stands for the quantity of labor allocated to the production of variety k. Since varieties are imperfect substitutes, each producer has some monopoly power and maximizes profits by charging a price

$$p^i = \frac{w^i}{\alpha} \; . \tag{4}$$

It is this equation which implies that all varieties produced in country i are equally priced. If the outcome of the model implies factor-price equalization, then all goods in the world economy have the same price.

The development of a new blueprint requires the allocation of a certain quantity of labor and some knowledge which is regarded as a public good (Romer, 1990). The production function reads $\dot{n}^i = L_R^i K_n^i$, where K_n^i represents the knowledge stock available for firms in country i and L_R^i is the quantity of labor of country i, which is allocated to R&D. Knowledge results as a by-product from the R&D activity and is assumed to be proportional to the number of differentiated goods available in the world economy; this amounts to saying that knowledge flows freely between countries. By a suitable choice of units, this relationship can be expressed by $K_n^i = n = n^{\mathrm{A}} + n^{\mathrm{B}}$. It should be noted that this assumption is crucial for the global-stability result. We will turn to this point in Sect. 5.

In analogy to static models free entry requires the absence of pure profits. Thus the present value v^i of the future profit stream resulting from the development of a new blueprint (and the subsequent production and selling of the variety) must equal its development costs and the free-entry condition yields, given that innovation takes place,

$$v^i = c_R^i = \frac{w^i}{n} \ . \tag{5}$$

In a perfect-foresight equilibrium, the value of the firm is determined by its future profits. We obtain

$$v^i(t) = \int_t^\infty \exp[-(R(\tau) - R(t))]\pi^i(\tau)\,\mathrm{d}\tau \ , \tag{6}$$

where profits $\pi^i(\tau) = (1-\alpha)p^i x^i$ are discounted by a cumulative factor $R(u) = \int_0^u r(s)\,\mathrm{d}s$ depending on the interest rate $r(s)$. Finally, the full-employment condition for the factor market requires that demand for labor of the R&D sector and of the production process equals fixed supply,

$$\frac{\dot{n}^i}{n} + n^i x^i = L^i \ . \tag{7}$$

It will be assumed throughout the paper that the labor force of country A is larger or equal to the one of country B, $L^{\mathrm{A}} \geq L^{\mathrm{B}}$.

3 Global Stability of the Balanced-growth Path

This section proves the global stability of the balanced-growth path for the above model. The proof establishes that country-specific innovation rates converge to a constant long-run value.

It is useful to introduce some definitions which allow to clarify expressions and to formulate the question asked in this paper most precisely. An equilibrium is the set of time paths of all endogenous variables that solves the above model. A solution satisfies all optimality conditions (among which are the transversality conditions), market-clearing conditions, and initial conditions for state variables. All time paths that are part of an equilibrium are called equilibrium paths. As will turn out later, there is a unique equilibrium for each pair of initial conditions and different initial conditions result in different equilibrium paths.

One special path is the balanced-growth path. It is defined by constant and internationally identical growth rates. It will be shown to be an equilibrium path. Since it will also be shown that the allocation of factors is time invariant, it will be called long-run equilibrium. All other equilibrium paths are short-run equilibria since factor allocation on these paths changes over time.

An equilibrium is called globally stable if there exist equilibrium paths for all initial conditions for state variables that converge to this equilibrium. Note that global stability of an equilibrium implies that this equilibrium is unique. The question of this paper is whether the BGP is globally stable and what the properties of short-run equilibrium paths are.

The structure of the proof of global stability is as follows: we show that the BGP is an equilibrium and derive relevant properties. By exploiting the structure of the model, we are able to show that some properties we find belonging to the BGP are also properties of any paths that lead to the BGP. This information simplifies the description of these equilibrium paths. Given this simplification, global stability of the BGP can be proven.

This section is structured accordingly. In Sect. 3.1, the BGP is described. On this path, firm values equalize internationally and growth rates of countries are constant and equal, as well. Section 3.2 shows that firm-value equalization is also a property of any path leading to the BGP, as long as both countries innovate.[3] Given this result on equilibrium paths, a reduced form that describes certain aspects of the model in terms of auxiliary variables can be easily analyzed (Sect. 3.3). By comparing findings about the auxiliary variables with properties of the balanced-growth path, it is straightforward to complete the proof.

3.1 Properties of the Balanced-growth Path

The BGP is defined by constant and internationally identical innovation rates,

$$\frac{\dot{n}^i}{n^i} = g \ . \tag{8}$$

Sufficient conditions (cf. appendix) for such a path to define an equilibrium are that firm values fall in both countries at the same rate as innovation takes place,

$$\frac{\dot{v}^i}{v^i} = -g \ . \tag{9}$$

No statement is made whether (9) are *necessary* conditions for the BGP to be an equilibrium since this is of no importance for the main proof.

3 Section 4.3 studies conditions under which this holds.

The following analysis suggests, however, that (9) is also necessary though no formal analysis is provided.

If (8) and (9) hold, the economy is on the long-run equilibrium BGP. Properties of such a path (cf. appendix) are internationally equal firm values,

$$v^{\mathrm{A}} = v^{\mathrm{B}} \equiv v \ , \tag{10}$$

and a world innovation rate, the rate at which new varieties are developed, of

$$g = (1-\alpha)L - \alpha\rho \ , \tag{11}$$

where $L = L^{\mathrm{A}} + L^{\mathrm{B}}$. The proof of global stability of the BGP will employ (8) and (9) and the equality of firm values on the BGP (10). Further generally interesting properties of the BGP, however, give useful insights or are used later for comparisons and are therefore reported here. The long-run innovation rate exceeds the sum of autarky innovation rates $g^i = (1-\alpha)L^i - \alpha\rho$ by $\alpha\rho$. The ratio of numbers of varieties $n^{\mathrm{A}}/n^{\mathrm{B}}$ to endowment is given by $n^{\mathrm{A}}/L^{\mathrm{A}} = n^{\mathrm{B}}/L^{\mathrm{B}}$ which makes clear why aspects of transitional dynamics are of importance in this model: this equation holds only by coincidence and everything that changes the necessary ratio of active firms puts the economy off its BGP. The value of all firms in the world economy is $V = vn^{\mathrm{A}} + vn^{\mathrm{B}}$ which equals the value of assets A held by consumers, $V = A = (L+\rho)^{-1}$. The value of firms in country i is $V^i = (L+\rho)^{-1}L^i/L$. Note that, as will be discussed in more detail in Sect. 4, it is not possible to determine accumulated savings A^i of a given country and therefore neither consumption levels nor trade patterns by looking at the BGP alone. Since the value of marginal product of labor in the R&D sector is equalized internationally factor-price equalization holds which implies by the price-setting behavior (4) that good prices equalize as well. Since further the value of firms falls at the same rate as firms enter the market, wages and prices are constant. Demand for a given variety and therefore the size of a firm can then be shown by (3) to fall at the innovation rate g.

3.2 Firm-value Equalization on Equilibrium Paths

A simple analysis of global-stability behavior and thereby of wages, consumption, trade patterns, and prices outside the BGP is feasible by exploiting interest-rate equalization due to international capital flows and the fact that dividend rates are a decreasing function of firm values. This second step represents the main innovation of this proof. It will be shown that firm values equalize internationally not only on the BGP (10) but on every equilibrium path that leads to the BGP. This step means – speaking more generally – that the ratio of two control variables is shown to be constant. This allows to reduce a differential equation system by one dimension (or more dimensions in more general cases, if more control variables can be shown to be fixed relative to another control variable). Such a one-dimensional reduction is sufficient here to obtain global results.

Consumers can finance R&D activities of firms in either country. Per-period returns they obtain from an investment v^i amount to the sum of capital gains plus dividends, $rv^i = \dot{v}^i + \pi^i$, an equation which can be obtained by differentiating (6) with respect to time. The per-period interest rate r is then given by the rate of capital gains $\dot{v}^i/v^i$ plus the dividend rate π^i/v^i. Since interest rates are equalized through free international capital flows, dividend rates can differ only if the rate of capital gains differ and that only in opposite direction: a lower dividend rate in, say, country A must be compensated for by a higher rate of capital gains, otherwise interest rates would not be equalized and all financial capital would flow to country B. Computing the rate of change of the relative firm value as

$$\frac{\dot{v}^{\mathrm{A}}}{v^{\mathrm{A}}} - \frac{\dot{v}^{\mathrm{B}}}{v^{\mathrm{B}}} = \frac{\pi^{\mathrm{B}}}{v^{\mathrm{B}}} - \frac{\pi^{\mathrm{A}}}{v^{\mathrm{A}}} \ , \tag{12}$$

shows, however, that differing dividend rates lead to a self-enforcing process if dividend rates are decreasing in firm values. Higher differences between dividend rates lead to higher differences in capital gains which in turn leads to higher differences in dividend rates, ad infinitum. Thus, if, say, $\pi^{\mathrm{B}}/v^{\mathrm{B}} > \pi^{\mathrm{A}}/v^{\mathrm{A}}$, the RHS of (12) is positive. Hence, v^{A} would grow faster than v^{B} and the gaps between dividend rates and between firm values would increase further and further. We know from the study of the BGP, however, that in the long-run equilibrium firm values equalize (10). Hence, if dividend rates decrease in firm values, firm-value equalization at the moment countries start to trade capital is a necessary condition for the economy to reach the BGP. In other words, firm-value equalization is the property of every equilibrium path.

The fact that dividend rates are indeed decreasing in firm values can be easily established for the present model by inserting (3), (4), and (5) into $\pi^i(\tau) = (1-\alpha)p^i x^i$, which gives

$$\frac{\pi^i}{v^i} = (1-\alpha)\frac{(v^i)^{-\varepsilon}}{n^{\mathrm{A}}(v^{\mathrm{A}})^{1-\varepsilon} + n^{\mathrm{B}}(v^{\mathrm{B}})^{1-\varepsilon}}E \ , \tag{13}$$

and differentiating. Hence, with (12) we can write $\dot{v}^{\mathrm{A}}/v^{\mathrm{A}} - \dot{v}^{\mathrm{B}}/v^{\mathrm{B}} \geq 0 \iff (v^{\mathrm{B}})^{-\varepsilon} - (v^{\mathrm{A}})^{-\varepsilon} \geq 0$. Therefore, firm values must equalize on every equilibrium path.

Note that Eq. (13) can be obtained only by using Eq. (5), the equality of firm values and entry costs. This equality holds only if a country has a positive innovation rate. If not, firm values fall short of entry costs. For the time being, positive innovation rates for both countries in the transition period are assumed. Section 4.3 will give a formal condition for country-specific positive innovation rates and a description of what happens if this condition does not hold.

3.3 Stability Analysis of a Reduced Form and Global Stability of the BGP

We will now use the result of firm-value equalization to derive a simple reduced form of the model. Choosing total expenditure $E = E^{\mathrm{A}} + E^{\mathrm{B}}$ as numeraire and setting it equal to unity implies, due to the assumption of internationally completely mobile financial capital, that nominal interest rates are given by the time-preference rate, $r = \rho$. We can then solve the factor-market clearing condition (7) for $\dot{n}^i/n^i$ and combine it with aggregate demand functions (3), where (4) and (5) and $v \equiv v^{\mathrm{A}} = v^{\mathrm{B}}$ have been inserted. With $n \equiv n^{\mathrm{A}} + n^{\mathrm{B}}$ and $\pi = (1-\alpha)/n$, the behavior of innovation rates is described by[4]

$$\frac{\dot{n}^i}{n^i} = \frac{n}{n^i} L^i - \frac{\alpha}{nv} , \tag{14}$$

$$\frac{\dot{v}}{v} = \rho - \frac{\pi}{v} . \tag{15}$$

The reduced form to be derived now describes the evolution of the two-country world in terms of aggregate country firm values. The idea, taken from Mulligan (1991) and Mulligan and Sala-i-Martin (1993), consists in transforming the variables of an original model, whose solution is a BGP, in such a way that the solution of the new model is a steady state (all variables are constant over time). We use as auxiliary

4 This specification implicitly rules out an overlap of varieties. Tang and Wälde (1996) show that trade can be welfare inferior to autarky if countries have partially produced identical varieties before opening up to trade and firms therefore have to compete for market shares in oligopolistic markets.

variables the values of all firms active in country A and B, respectively,[5]

$$V^{\mathrm{A}} = n^{\mathrm{A}} v, \quad V^{\mathrm{B}} = n^{\mathrm{B}} v \ . \tag{16}$$

Some rearrangements, starting from (14) and (15), give the reduced form,

$$\dot{V}^{\mathrm{A}} = (V^{\mathrm{A}} + V^{\mathrm{B}})L^{\mathrm{A}} + \rho V^{\mathrm{A}} - \frac{V^{\mathrm{A}}}{V^{\mathrm{A}} + V^{\mathrm{B}}} \ ,$$
$$\dot{V}^{\mathrm{B}} = (V^{\mathrm{A}} + V^{\mathrm{B}})L^{\mathrm{B}} + \rho V^{\mathrm{B}} - \frac{V^{\mathrm{B}}}{V^{\mathrm{A}} + V^{\mathrm{B}}}, \quad V^{\mathrm{A}}, V^{\mathrm{B}} > 0 \ .$$

It is obvious that if the two-country world is on a BGP where, according to (8), n^{A} and n^{B} grow at g and v falls at g, the sums of firm values in either country (16) are constant. Hence, a necessary condition for global stability of the BGP is convergence of V^{A} and V^{B} to constant values. The task in this second step therefore simply consists in studying this reduced form and showing that V^{A} and V^{B} are constant in the long run. To this end, we simply have to find the zero-motion loci and can then undertake a phase-diagram analysis in the $(V^{\mathrm{A}}, V^{\mathrm{B}})$-space. The zero-motion loci $(V^{\mathrm{A}}, V^{\mathrm{B}})$ for $\dot{V}^{\mathrm{A}} = 0$ are given by

$$(V^{\mathrm{A}} + V^{\mathrm{B}})L^{\mathrm{A}} + \rho V^{\mathrm{A}} - \frac{V^{\mathrm{A}}}{V^{\mathrm{A}} + V^{\mathrm{B}}} = 0 \ . \tag{17}$$

In order to get an idea of where these loci can be found in the positive quadrant (firm values cannot become negative), we determine the intersection point of these loci with rays starting from the origin, given by $V^{\mathrm{B}} = \theta V^{\mathrm{A}}$, where θ ranges from zero to (plus) infinity. Inserted into (17), the intersection points are at

$$(1+\theta)V^{\mathrm{A}}L^{\mathrm{A}} + \rho V^{\mathrm{A}} - (1+\theta)^{-1} = 0$$
$$\iff V^{\mathrm{A}} = (1+\theta)^{-1}[(1+\theta)L^{\mathrm{A}} + \rho]^{-1} \ .$$

5 As noted by a referee, other auxiliary reduced forms can be used. One possibility is to consider the ratio of varieties $\eta \equiv n^{\mathrm{B}}/n^{\mathrm{A}}$ and the value of all firms in the world economy $V = (n^{\mathrm{A}} + n^{\mathrm{B}})v$. Using these two auxiliary variables is advantageous for the present model. The more general approach in the main text, especially the use of a phase-diagram analysis below, however, has proved useful in another context (Tang and Wälde, 1996) as well.

Thus the zero-motion loci are given as a function of θ as

$$(V^{\mathrm{A}}, V^{\mathrm{B}}) = \left(\frac{(1+\theta)^{-1}}{(1+\theta)L^{\mathrm{A}}+\rho}, \frac{\theta(1+\theta)^{-1}}{(1+\theta)L^{\mathrm{A}}+\rho}\right) .$$

Given this notation, we see that there is one intersection point with the V^{A}-axis for $\theta = 0$ at $((L^{\mathrm{A}} + \rho)^{-1}, 0)$. Letting θ increase, V^{A} decreases and approaches zero as θ goes to infinity. V^{B}, however, is not monotonic in θ. Since the slope of V^{B} at $\theta = 0$ is positive it increases first and then at a certain point must decrease since – apply L'Hôpital's rule – V^{B} approaches zero as θ goes to infinity. Thus we can unambiguously draw the zero-motion loci determined by (17) as in Fig. 1. In an analogous way the zero-motion loci for $\dot{V}^{\mathrm{B}} = 0$ can be determined. Note that we did not make any statements whether the maxima of the zero-motion loci (with respect to V^{A} and V^{B}, respectively) are to the left or to the right of the intersection point of zero-motion lines. This, however, is of no importance for the stability properties of the system and can thus safely be ignored.

Let us now return to the original underlying model. Assume that two countries begin to trade with each other which are of different size. In the context of the present model, this means that the country with a larger resource base, country A, has a higher autarky innovation rate than the other country. As long as both countries do not trade, the lead of country A over country B will continuously expand and the ratio of differentiated goods will steadily increase and deviate further and further from its trade-BGP value $n^{\mathrm{A}}/n^{\mathrm{B}} = L^{\mathrm{A}}/L^{\mathrm{B}}$. The question we want to answer is: Is there an initial value v for every initial ratio of differentiated goods $n^{\mathrm{A}}/n^{\mathrm{B}}$ such that the two-country world finds itself on the saddle path in Fig. 1 and thus converges to the equilibrium point F?[6]

Since history dictates initial conditions for n^{A} and n^{B}, the ratio of aggregate country firm values V^{A} to V^{B} is equally given by history and is represented in Fig. 1 by a dashed ray starting from the origin and denoted by OR. The bigger the advance of country A over country B in terms of differentiated goods, the flatter the ray OR and vice versa. Now the question of global stability is equivalent to asking whether a firm

6 The question might arise how the appropriate value v is found by agents in this economy. The value of a firm is not as intuitive a control variable as the more usual level of consumption. The system can be reformulated by choosing another numeraire, which would result (if the firm value were the numeraire) in total world expenditure to be the control (choice) variable of economic agents. Hence, it is justified to think of v as a variable that is determined by the consumption–saving choice of households.

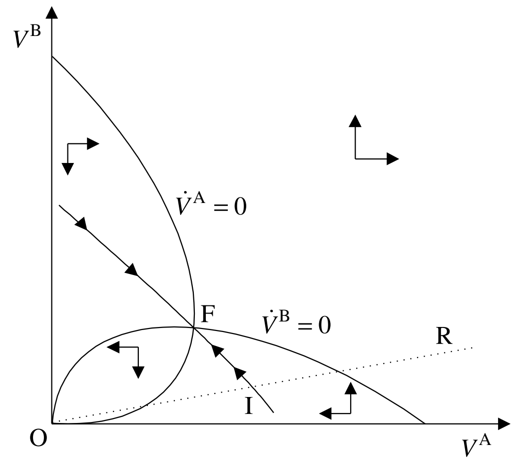

Fig. 1: Global saddle-path stability of the reduced form

value exists such that initial conditions for V^{A} and V^{B} always lie on the saddle path (point I). Showing its existence, however, is straightforward. Varying v means moving up and down on OR, thus v has simply to be chosen by agents in this economy such that initial conditions are indeed given by point I. Thus, independently of the initial quantities of differentiated goods in country A and country B, the world economy always converges to an equilibrium where aggregate country firm values V^{i} are constant. The reduced form is globally saddle-path stable.[7]

Completing the proof of global stability of the model's BGP is now straightforward. The stability analysis in Fig. 1 has shown that the values of all firms in a country converge to a constant long-run value. By (16) we know that this implies that the number of firms in a country grows at the same rate as the value of a representative firm falls, $\dot{n}^{i}/n^{i}=-\dot{v}/v$. This further establishes that the number of firms in country A grows at the same rate as the number of firms in country B, $\dot{n}^{A}/n^{A}=\dot{n}^{B}/n^{B}$. It has been argued in Sect. 3.1 (and shown formally in the appendix) that imposing these equalities on a solution of the model implies that the economy finds itself on the long-run equilibrium BGP, where the innovation rate is given by (11). Hence, convergence to the long-run equilibrium of the reduced form is identical to convergence to the BGP. It thereby has been established that innovation rates of either

7 A local stability analysis of a linearized version of Eqs. (6) differentiated with respect to time and (7) transformed to yield a stationary solution shows that the system is locally saddle-path stable (Wälde, 1995, chap. 4).

country converge to the long-run innovation rate (11). The BGP of the model is globally stable.

4 Properties of the Transition Period and Implications for the Long Run

The last section has presented a proof of global stability of the BGP under the condition that both countries continue to innovate under free trade. This has established the convergence of innovation rates of both countries in the long run. This section studies properties of equilibrium paths and implications for long-run values of several variables. It also studies circumstances under which the condition of positive country-specific growth rates holds.

Section 4.1 looks at the behavior of wages and firm size on equilibrium paths and highlights the economic mechanism behind convergence. Section 4.2 looks at the behavior of innovation rates of countries during the transition period. Using results obtained there, Sect. 4.3 studies the circumstances under which a zero-growth period is relevant and thereby completes the proof of Sect. 3. In Sect. 4.4, we show that consumption levels are path dependent, that convergence of GDP is not instantaneous and that trade balances reverse during the adjustment period. Finally, a closed-form solution of the model is derived, by which a measure for the speed of convergence of GDP can be presented and the behavior of the world economy can be easily illustrated (Sect. 4.5).

4.1 Wages, Firm Size, and the Catching-up Mechanism

The last section has shown that, given positive innovation rates for both countries, both the transition period and the BGP are characterized by an equalization of firm values. Due to perfect international knowledge spillovers, the value marginal product of labor in the R&D sector equalizes which implies that factor-price equalization carries over to the transition period as well. Since then all firms face identical production costs, all varieties are equally priced (4) which in turn implies by (3) that the firm size equalizes across countries. Nominal factor and good prices are constant since the decrease of the output value of labor in the R&D sector is compensated by the increase in the productivity of labor due to the increase of knowledge available for R&D. The plant size decreases by (3) as new competitors enter the market. This establishes a general finding on factor prices or prices in general: BGP properties directly carry over to the equilibrium path converging towards the BGP, given firm-value equalization on equilibrium paths as explained in Sect. 3.2 and internationally constant labor-productivity ratios. Hence, productivity of labor must change in both countries at the same rate, which means in the present model perfect international knowledge spillovers.

These properties of the equilibrium paths exclude that the catching-up process is driven by an R&D cost advantage. R&D costs are given by (5), factor rewards divided by the domestic knowledge stock. Both quantities are independent of the country, however, given free international flows of capital and knowledge. In order to explain catching-up, other reasons have therefore to be evoked why in the lagging country more factors are allocated to the R&D sector. Looking at the factor-market clearing condition (7) shows that the number of factors in the production sector increases in the number of firms active in that country. Since plant size equalizes internationally, the country that has more labor per firm [divide (7) by n^i] has more factors available for the R&D process and therefore a higher innovation rate. Since this is the case for the laggard, convergence in innovation rates takes place.[8]

4.2 Innovation Rates During the Transition Period

A further interesting implication of firm-value equalization is that in the world as a whole the rate at which new goods are developed is constant all during and after the transition period. This allows to compute country-specific innovation rates. Start by adding budget constraints of both countries given by (2) which gives, by using (5) and $v^{\mathrm{A}} = v^{\mathrm{B}} \equiv v$, $E + \dot{A} = \rho A + nvL$. Divide by world assets $A = A^{\mathrm{A}} + A^{\mathrm{B}}$, observe that $A = nv$, $E = 1$, and $r = \rho$, to obtain a world budget constraint which reads

$$\frac{\dot{A}}{A} = \rho + L - \frac{1}{A} \ . \tag{18}$$

We know from the analysis of the BGP that in the long run the value of all firms in the world economy is given by $V = (L + \rho)^{-1}$. Since this value has to equal the value of world assets, the long-run equilibrium value of accumulated world savings A is given by that value, too.

8 An immediate implication is that convergence as observed here can take place only if the R&D sector can carry out the "buffer-stock" function sketched above. One condition is that labor moving from the production sector is instantaneously capable of acquiring all the skills necessary for doing the research task, which is a reasonable assumption only as a first approximation.

Equation (18), however, tells us that this equilibrium value can only be reached if total wealth is constant at every moment in time and given by $A = V = (L+\rho)^{-1}$. Since the value of world assets is $A = vn$, the value of the firms v therefore *always* falls at the same rate as the number of varieties in the world n increases. Computing the world innovation rate then shows that this can hold only if it is constant and equal to the BGP innovation rate g as given in (11). Thus, though innovation rates in countries differ during the adjustment period, new varieties are always introduced, in the world as a whole, at the constant rate of g.

But how do innovation rates of countries look like? In autarky, the innovation rate of the larger country is higher which is shown in Fig. 2 for the period before t_{trade}. Then, during the adjustment period, the innovation rates of country A and country B approach the BGP innovation rate asymptotically. Given the definition of V^{A} and V^{B}, the innovation rates of country A and B are

$$\frac{\dot{n}^{\text{A}}}{n^{\text{A}}} = \frac{\dot{V}^{\text{A}}}{V^{\text{A}}} - \frac{\dot{v}}{v} = \frac{\dot{V}^{\text{A}}}{V^{\text{A}}} + g \quad \text{and} \quad \frac{\dot{n}^{\text{B}}}{n^{\text{B}}} = \frac{\dot{V}^{\text{B}}}{V^{\text{B}}} - \frac{\dot{v}}{v} = \frac{\dot{V}^{\text{B}}}{V^{\text{B}}} + g \ ,$$

respectively. Since by the stability analysis we know that V^{A} decreases whereas V^{B} increases, given that country A has developed more varieties relative to its labor endowment than country B, $\dot{V}^{\text{A}}/V^{\text{A}}$ is negative and $\dot{V}^{\text{B}}/V^{\text{B}}$ is positive. Therefore, the innovation rate of country A is always smaller than the long-run innovation rate g, whereas the innovation rate of country B "overshoots" and approaches its long-run value from above.

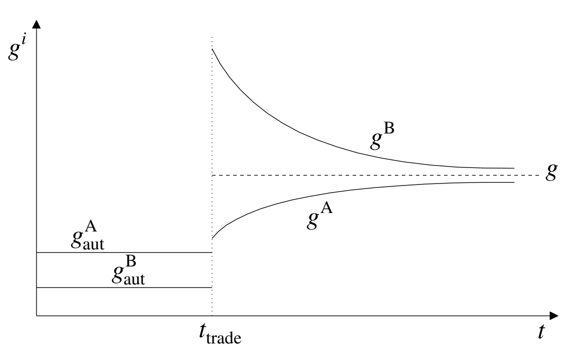

Fig. 2: Innovation rates before and after opening up to trade

4.3 Conditions of the "Zero-growth" Period

One step in the proof of global stability [the derivation of Eq. (13)] required that innovation rates of both countries do not become negative during the transition period. Given that the results obtained so far depend on this assumption, we now study the circumstances under which this assumption holds. Again, it is the fact that the world innovation rate is constant during the entire transition period, derived in Sect. 4.2, which allows to derive an explicit expression. We focus on the leading country A and undertake the same reasoning about its innovation rate during the adjustment period as we did for the backward country in Sect. 4.1 when the catching-up mechanism was studied. It is clear that a low labor-per-firm ratio implies that factors move out of the R&D into the production sector which implies that the growth rate is the lower the lower this per-firm endowment. Obviously, the interesting question is: can there be a period of "zero growth"? Can the innovation rate $g^{\mathrm{A}}(t_{\mathrm{trade}})$ be lower than depicted in Fig. 2 or even become zero for some time[9]?

Looking again at the factor-market clearing condition (7) shows that the innovation rate equals zero if labor endowment is too small to satisfy demand by the production sector, $g^{\mathrm{A}} = 0 \iff L^{\mathrm{A}} < n^{\mathrm{A}}x$. Inserting $nv = A = (L + \rho)^{-1}$ in (14), shows after rearranging and inserting the world innovation rates that this is equivalent to

$$g^{\mathrm{A}} = 0 \iff \frac{n^{\mathrm{A}}/L^{\mathrm{A}}}{n^{\mathrm{B}}/L^{\mathrm{B}}} \gtrless \frac{L^{\mathrm{B}}}{L^{\mathrm{B}} - g}, \quad \text{if} \quad L^{\mathrm{B}} \gtrless g \; . \qquad (19)$$

This condition requires to distinguish two cases. For understanding this distinction note that the world innovation rate g equally amounts to the labor force allocated to R&D in the world economy as a whole. The first case, $L^{\mathrm{B}} > g$, considers a parameter constellation where the number of workers in country B exceeds the number of researchers in the world as a whole. This is certainly the normal case. The second case is the one of a very small country B; that small that the labor force in this country is smaller than there are researchers in the world, $L^{\mathrm{B}} < g$.

The zero-growth condition (19) is illustrated in Fig. 3. The number

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9 The innovation rate cannot become negative since this would require firms to exit the market. This is excluded since firms always make positive profits. Note that we cannot find an *upper* bound for $g^{\mathrm{A}}(t_{\mathrm{trade}})$ since the closer the two countries are to their common BGP, the higher is the innovation rate of country A. The upper bound for country B is clearly given by its factor endowment.

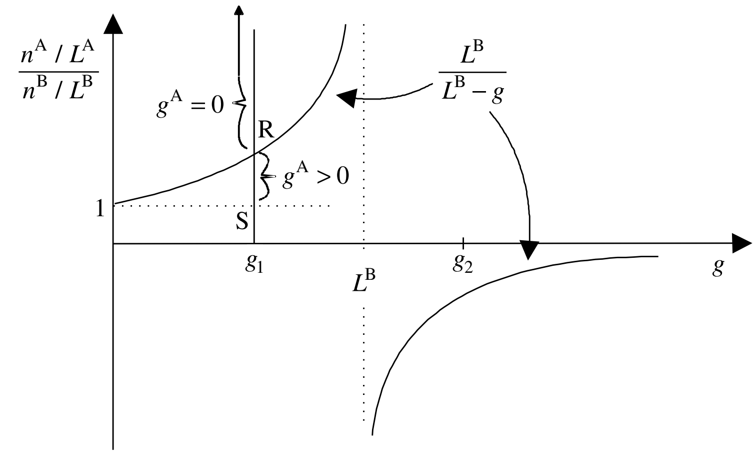

Fig. 3: The zero-growth condition

of researchers in the world is plotted on the horizontal axis. The vertical axis shows relative firm-per-capita endowment. This ratio lies between unity (its BGP value) and infinity. It must exceed unity since country A is the larger country by assumption and therefore continuously extends its lead over country B as long as countries are in an autarky situation where they experience no international knowledge spillovers. The ratio on the RHS of (19), $L^B/(L^B - g)$, can be seen as a measure for how far the relative firm-per-capita endowment $\frac{n^A/L^A}{n^B/L^B}$ may exceed its BGP ratio (which is unity) before zero growth in the leading country A sets in.

We start with the normal case, where the number of researchers world-wide is smaller than the size of the labor force of country B. Assume the world innovation rate is g_1, as shown in Fig. 3. The ratio of firms to labor endowment $\frac{n^A/L^A}{n^B/L^B}$ lies above S. If the deviation from unity is too big, hence above $L^B/(L^B - g)$, denoted by R in Fig. 3, country A stops innovating at t_{trade}. Innovation restarts when the increase of the number of firms in country B has decreased the LHS of (19) below R. Clearly, the probability of a zero-growth period is reduced the higher world employment in R&D, hence the higher the world innovation rate g_1; equivalently, the bigger country A, the more consumers value different varieties (low α) and the more patient they are (low ρ).

If country B is very small, the second case, illustrated by g_2 in Fig. 3, becomes relevant. In such a setting, a zero-growth period would never set in simply because even if all labor of country B were allocated towards catching up, this would not be enough to match the rate at which new firms enter the world economy as a whole. Clearly, such a situation can be imagined only if country A begins to trade with a much smaller country.

The prediction of a zero innovation period for the leading country is a rather extreme one but is a logical consequence of the assumption of perfect international capital flows. If firms are internationally very unequally distributed in per-capita terms, which is similar to unequal capital-per-labor ratios, all new investment takes place in the laggard country. Hence, if the world economy is too far away from its BGP, i.e., if country A is too advanced, all innovation will be concentrated in country B. If one relaxed the assumption of perfect international capital flows, however, hence if some savings of the leading country were not reinvested in the foreign country but were available domestically for R&D investments, a zero-innovation period would be excluded even for very unequal initial conditions.

Due to the special character of a zero-growth period, we sketch only briefly how a detailed analysis could be carried out and what happens during such a first phase of transition: the wage rates of trading partners are related by country A's factor-market clearing condition (7). Since the number of firms producing in country A is constant, this relation is static. Further relevant equations are then the factor-market clearing equation for country B which can be solved for the innovation rate, and an equation, derived from (5) and (6), which determines the evolution of wages in country B over time. We therefore have a reduced form consisting of one static equation determining w^{A}, and two differential equations determining n^{B} and w^{B}. Since it can be shown that innovation in country B must be positive and that wages in the backward country are lower than in the leading country as long as innovation stands still, this period is characterized by a convergence of wage rates and ends with factor-price equalization exactly at the point where innovation in the leading country A resumes. We therefore have a similar result to standard trade theory, though with a completely different background: if both countries are incompletely specialized, factor prices equalize, otherwise they do not.

4.4 Consumption Levels, Convergence of GDP, and the Trade Balance

Solving the dynamic budget constraint (2) for asset holdings $A^i(t)$ of households of country i yields, by taking the transversality condition into account, the familiar intertemporal budget constraint. It limits the present value of future expenditure at any point in time t by the sum of wealth and the present value of future income,

$$A^i(t) + \int_t^\infty \exp\Big[-\int_t^\tau r(s)\,\mathrm{d}s\Big] w^i(\tau) L^i \,\mathrm{d}\tau$$
$$= \int_t^\infty \exp\Big[-\int_t^\tau r(s)\,\mathrm{d}s\Big] E^i(\tau)\,\mathrm{d}\tau \ .$$

Employing the fact that on equilibrium paths nominal interest rates, wages, and expenditure levels are constant, $r = \rho$, $w^i = \bar{w}^i$, $E^i = \bar{E}^i$, the economies' budget constraints read

$$A^i = \rho^{-1}(\bar{E}^i - \bar{w}^i L^i) \ .$$

It shows that A^i is constant on equilibrium paths as well. This value is determined by two components: the number of firms active in a country, which is entirely predetermined by the number of firms active in the economy before capital flows set in, and the value of firms after trade in capital is allowed for, $A^i = v^i n^i$. At the moment trade in capital is allowed for, a revaluation of all firms takes place but no firm shares are traded internationally before new firm entries require financing of their R&D expenditures.[10] Hence, though international capital flows are allowed for, no *instantaneous* capital flows set in. As a consequence, relative wealth is a function solely of the number of active firms prior to trade in capital. The fact that A^i is constant and given by history in this setup of perfect international capital flows has interesting implications for consumption levels, convergence of GDP, and the trade balance to which we now turn.

Since wages are equalized internationally, the level of consumption of a country E^i depends entirely on the value of assets of that country A^i after trade in financial capital is opened up. If both countries, before taking up trade in capital, have the same ratio of firms to labor endowment (by, say, having already enjoyed knowledge spillovers for a while) and therefore are already on their common BGP, asset holdings will be given by $A^i = (L+\rho)^{-1} L^i/L$ and the expenditure level of a country i is $E^i = L^i/L$. In this case, consumption per capita is equalized. Considering the general case, where countries are not on their common BGP immediately after starting to trade in capital, expenditure per capita will never equalize as a result of international borrowing and lending. What we find in the latter case is the well-known consumption-smoothing effect due to the possibility of international lending and borrowing: the

10 I am indebted to Elhanan Helpman for discussion of this point.

expenditure level of both countries is constant but for the backward country B on a higher level – compared to a situation with *no* international capital flows – at t_{trade} but lower in the long run. Expenditure of the leading country, however, is initially lower but will in the long run be higher than the no-capital-flow level.

A further consequence of the absence of instantaneous capital flows is slow convergence of GDP. This implication stands in stark contrast to but is more plausible than models with international physical capital flows where capital is a factor of production. In the latter model GDP per capita instantaneously equalizes due to a jump of debt D^{B} of the poorer country, here country B, to equalize the value of marginal products:

$$Y_{K^{\mathrm{A}}}(A^{\mathrm{A}} - D^{\mathrm{B}}, L^{\mathrm{A}}) = Y_{K^{\mathrm{B}}}(A^{\mathrm{B}} + D^{\mathrm{B}}, L^{\mathrm{B}}) \ , \tag{20}$$

where $K^i = A^i + D^i$. It is this implausible prediction of instantaneous capital flows which led many authors to introduce explicit adjustment costs to obtain smooth behavior (e.g., Lipton and Sachs, 1983; Matsuyama, 1987; Sen and Turnovsky, 1989). The present model does not predict instantaneous capital flows even in the absence of adjustment costs since, as argued above, allowing for capital flows leads to a revaluation of firms only and debt is zero at t_{trade}. Afterwards, debt of the backward country B ($D^{\mathrm{B}} = -D^{\mathrm{A}}$) accumulates only slowly due to the trade-off between savings in the form of R&D and production. More formally, this can be seen by considering the identity $A^{\mathrm{B}} = V^{\mathrm{B}} - D^{\mathrm{B}}$. Asset holdings of each country are constant and the value of firms of country B approaches its long-run value from below: when V^{B} increases and A^{B} is constant, D^{B} increases. As debt increases and new firms are created, GDP converges.

The trade balance is known to be nothing but a residual to intertemporal consumption and saving decisions as emphasized by the literature on current-account dynamics (e.g., Obstfeld, 1982; Svensson and Razin, 1983; Persson and Svensson, 1985). In models with physical capital, debt of the poorer country starts from an initial level determined by (20). After the initial jump, it slowly increases to reach its long-run steady-state level.[11] The trade balance *TB* of the leading country is al-

11 Perfect international capital flows lead to an equalization of the per-capita capital stock k given linearly homogeneous production functions. Optimal per-capita consumption c^i accumulation paths are given by $c^i = \rho(h + a^i)$ (Blanchard, 1985; with probability of death $p = 0$), where h is the sum of discounted future per-capita labor income which is identical for both countries since interest rates and wages are equalized and a^i are per-capita asset holdings which equal the difference between domestic capital stock and debt, a^i

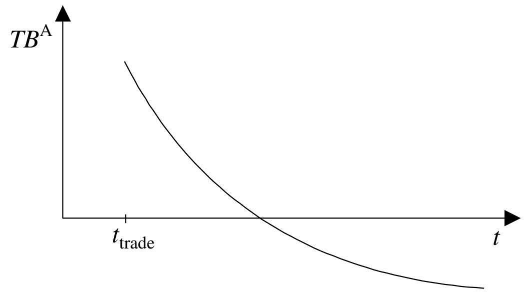

Fig. 4: The trade balance of the leading country A

ways negative.[12] This reflects income from foreign wealth and slow international capital flows. The behavior of the trade balance in the present model can be found by considering country A's Balance of Payments identity, $TB^A = \dot{D}^B - rD^B$. Since initially foreign holdings of country A, which is indebtedness of country B, are zero but there is a positive flow of financial capital to country B, the trade balance of country A is positive. This reflects lending of part of domestic production of country A to country B. After some time, however, when income from foreign wealth rD^B exceeds the flow of foreign R&D financing, the trade balance reverses and becomes negative. In the long run, no further debt is accumulated and the trade balance of the rich country is negative since the leading country receives income from firms it owns abroad. The time path of the trade balance is illustrated in Fig. 4.[13]

The prediction that a certain fraction of firms in the poor country are owned by inhabitants of the rich country can be made more precise. With constant wealth A^B and values of firms falling at the world

$= k - d^i$. Since $\dot{c}^i/c^i = r - \rho$, consumption of both countries increases at the same rate and $(\dot{h} + \dot{k} + \dot{d}^B)/c^A = (\dot{h} + \dot{k} - \dot{d}^B)/c^B$. Solving this for $\dot{d}^B$ shows $\dot{d}^B > 0$ since by assumption $a^A(\infty) > a^A(t_{trade}) > a^B(t_{trade})$ and therefore $c^A > c^B$.

12 The trade balance of a country is given by the difference of gross domestic product, consumption, and domestic investment. Observing the equalization of per-capita GDPs and capital stocks this reads $TB^i/L^i = y - c^i - \dot{k}$. Since $TB^B = -TB^A$, subtracting yields $TB^A/L^A + TB^A/L^B = -c^A + c^B$ which implies $TB^A < 0$.

13 A trade-balance reversal was also found by Backus et al. (1992). There, the effect is due to a sequence of exogenous disturbances caused by an international transmission of technology shocks. Here, the reversal of the trade balance takes place as a result of one unique exogenous event, the opening up to trade.

innovation rate g, the number of firms owned by households in country B increases at g, $n^{\mathrm{B}}_{\text{B-owned}}(t) = n^{\mathrm{B}}_{t_{\mathrm{trade}}} \exp[gt]$, $t > t_{\mathrm{trade}}$. This implies that R&D of the "extra" firms created during the catching-up process in country B is entirely financed by households of country A. Therefore ownership of those firms lies in country A, $n^{\mathrm{B}}_{\text{A-owned}}(t) = n^{\mathrm{B}}(t) - n^{\mathrm{B}}_{t_{\mathrm{trade}}} \exp[gt]$, where $n^{\mathrm{B}}(t_{\mathrm{trade}}) = n^{\mathrm{B}}_{t_{\mathrm{trade}}}$.

4.5 A Closed-form Solution, Halflife of Convergence, and the Saddle Plane

The finding that the world innovation rate is constant all during the transition period allows to carry the analysis of the adjustment period even further, since, given this property, one can compute the closed-form solution of the model. This solution can be used to derive a measure for the speed of how fast GDPs per capita of two countries converge. The finding of slow convergence in the last section can thereby be formulated more precisely. One can further nicely illustrate the dynamic behavior of the entire world economy by computing the saddle path, equilibrium paths, and the saddle plane and plotting them in a figure.

To this end, insert $A = nv = (L + \rho)^{-1}$ into (14) and solve the resulting *linear* differential equation system $\dot{n}^i = (n^{\mathrm{A}} + n^{\mathrm{B}})L^i - n^i \alpha(L + \rho)$. Eigenvalues are the world innovation rate g and $\lambda \equiv -\alpha(L + \rho)$. Computing eigenvectors, the (general, with constants c_1 and c_2 determined by initial conditions) closed-form solution reads[14]

$$\begin{pmatrix} n^{\mathrm{A}} \\ n^{\mathrm{B}} \end{pmatrix} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} \exp[\lambda t] + c_2 \begin{pmatrix} L^{\mathrm{A}} \\ L^{\mathrm{B}} \end{pmatrix} \exp[gt] \ . \qquad (21)$$

A natural measure of the speed of convergence of GDP per capita of two countries is the halflife of the difference of their firm-per-capita ratios, $n^{\mathrm{A}}/L^{\mathrm{A}} - n^{\mathrm{B}}/L^{\mathrm{B}}$. The more these ratios differ, the bigger are differences in output per capita of these countries. In the long run, where these ratios equalize internationally, GDP per capita equalizes. This convergence over time is due not only to the fact that capital is not a factor of production, as stressed in the last section, but also because

14 The fact that there is a closed-form solution for the above system could have been used earlier which would have made the third step of the proof of global stability redundant (the stability analysis of a reduced form). This step, however, is more general than applying (21) and is therefore more flexible for application in other cases. In fact, its usefulness for other models has proven in Tang and Wälde (1996).

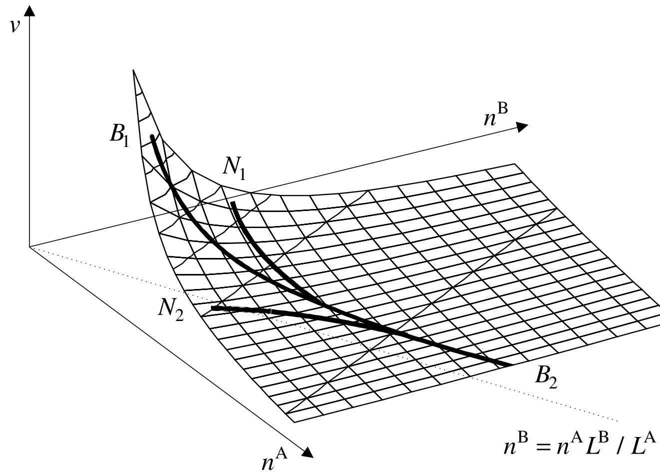

Fig. 5: Saddle plane, equilibrium paths, and the balanced-growth path

of the implicit assumption of irreversibility of investment in firms or, in other words, immobility of firms across borders.

Generally speaking, the halflife of a process $a(t)$ is the time $T - t$ that is required to reduce $a(t)$ by 50%, implicitly defined by $a(T) = a(t)/2$. Assuming that $a(T)$ falls exponentially at a rate of $\lambda < 0$, the halflife is given by $\ln(2)(-\lambda)^{-1}$. By employing the closed-form solution it can be shown that the distance between the measures of GDP per capita falls at a rate equal to the negative root of (14), given by $\lambda \equiv -\alpha(L + \rho)$. Halflife of convergence is therefore given by $\ln(2)(\alpha(L + \rho))^{-1}$. Theoretically predicted convergence of GDP per capita is therefore the faster the bigger the countries which trade with each other and the higher the time-preference rate or the higher the elasticity of substitution between varieties $\varepsilon = 1/(1 - \alpha)$. Interestingly, the argument is not simply the faster countries innovate, the faster they converge, since high ρ and high α reduce the world innovation rate. What should be interpreted with care is the impact of the size of countries; this term would appear differently if knowledge spillovers had not been assumed to be perfect. Consequently, it would be misleading to argue that regions of highly populated areas should converge (neither grow) faster than small trading blocs, simply because the condition of perfect knowledge spillovers (apart from other conditions) is not fulfilled. It is finally worthwhile to recall that convergence in GDP per capita does not mean convergence in income per capita due to the possibility of international borrowing and lending. Section 4.4 takes up on this point.

The closed-form solution further allows to compute the saddle plane of the world economy which is the set of all equilibrium paths. It is described by values of the representative firm v as a function of the number of firms in the world economy. Since the value of all firms A is known, the saddle plane is simply $v = A/n$. This plane is plotted in Fig. 5. The horizontal axes show the number of firms in country A and B, respectively, and the vertical axis measures the value of the representative firm v. The BGP of the world economy lies on the saddle plane and is denoted by B_1B_2. It is a projection of the BGP property $n^A/L^A = n^B/L^B$, drawn as the dotted line in the (n^A, n^B) space in Fig. 5, into the saddle plane.

When two countries start trading, initial conditions are described by the state variables n^A and n^B. By an optimal adjustment of consumption levels, a firm value v results, such that the economy jumps on the saddle plane. Once there, the economy finds itself on equilibrium paths which converge to the BGP. Figure 5 contains two examples of equilibrium paths. One that is characterized by a firm ratio n^A/n^B that lies below its long-run value (starting at N_1) and one that lies above (starting at N_2). Given these initial conditions, the economy finds itself at N_1 or N_2 and moves, as the number of varieties in the world increases, towards B_2.

5 Discussion of the Model and Possible Extensions

In order to keep the analysis tractable, the results of the previous sections have been derived in a model deliberately chosen to be as simple as possible. The crucial finding which facilitates a proof of global stability is the fact that the dividend rate must be equalized internationally. In the present model this led to an equalization of firm values, which simplified but by no means is necessary for the proof of global stability. This convenient implication holds for other models of that type, e.g., where economies produce both a differentiated and a homogeneous good with several factors of production (Grossman and Helpman, 1991a, chap. 7), where long-run growth peters out (Grossman and Helpman, 1989), or where time-preference rates differ internationally (Wälde, 1994a, b) and deriving global findings would be straightforward. In the latter case, however, the share in world expenditure of the more patient country would approach unity, whereas the other country's share would fall. In a model where one country has a comparative advantage in R&D (Grossman and Helpman, 1990) firm values (and therefore factor prices and firm values) do not equalize, but dividend ratios would equalize on the saddle path. This then fixes the ratio of firm values (to a value different from unity) which leads to constant relative wages and prices all along the equilibrium path towards the BGP. A model with internationally differing technologies would not predict a constant world innovation rate g but a reasonable conjecture of its time path can be made by taking the reallocation mechanism of labor into consideration which takes place at t_{trade}: the backward country shifts resources to the R&D sector whereas the leading country specializes temporarily in the production of goods. Assuming the leading country is characterized by a comparative advantage in R&D then the transition period is characterized by a lower world innovation rate which approaches its long-run value from below. Such a model is also characterized by slow accumulation of foreign debt and path-dependence of long-run per-capita consumption levels. Country innovation rates would behave qualitatively in the same way as in the simple model above.

A counter-example where an easy stability analysis as sketched above is not possible is the model of endogenous product cycles by Grossman and Helpman (1991b): a two-country world is characterized by R&D activities in a technologically advanced[15] northern country and imitation of varieties developed in the north by a southern country. The values of firms equal the discounted sum of future profits; in the south firm profits are discounted at the international interest rate whereas in the north this interest rate is augmented by the probability of being imitated μ to adjust for the risk that firms stop production and distribution of dividends. Since the northern no-arbitrage condition reads $\dot{v}^N/v^N = -\pi^N/v^N + r + \mu$, the change of the ratio of firm values over time becomes a function of the imitation rate which excludes a simple stability analysis.

The analysis above focused on the case of perfect international knowledge flows, as expressed by $K_n^i = n = n^{\mathrm{A}} + n^{\mathrm{B}}$. This can be viewed as one benchmark case compared to the other extreme of domestic spillovers, $K_n^i = n^i$, whereas inbetween there is a continuum of cases with decreasing returns to foreign knowledge, $K_n^i = n^i + (n^j)^{\varepsilon}$, $\varepsilon < 1$, or time lags in the dissemination of knowledge (Grossman and Helpman, 1991a, chap. 3). The case of spillovers restricted to the economy of origin was analyzed in Grossman and Helpman (1991a, chap. 8) where it is shown that convergence in innovation rate is excluded and R&D will be undertaken (apart from a purely coincidental case) in one country only. The case of decreasing returns to foreign knowledge or time lags can be regarded as a generalization of domestic spillovers and implications are similar: the country with a higher knowledge stock when opening up to trade will have a productivity advantage

15 In the sense of a higher productivity of labor in the R&D sector but identical productivity in the production sector.

in R&D and will thereby innovate and also increase its knowledge stock faster than the other country. Even if after some time some knowledge will be disseminated internationally and the spillovers between countries are not asymmetric (i.e., the backward country can adopt all the knowledge from the leading country but not vice versa) the lead of the high-knowledge country will be ever expanding and no convergence in innovation rates takes place. This means that the benchmark case of purely domestic knowledge represents all the intermediate cases of imperfect international knowledge spillovers. Note that in contrast it is not to be expected that *proportional* international knowledge spillovers, $K_n^i = n^i + \varepsilon n^j$, would lead to these strong hysteresis effects but one would rather find convergence to a common long-run innovation rate which, however, would be lower than in the case of perfect spillovers. A detailed analysis of imperfect knowledge spillovers will be the focus of future research.

6 Conclusion

This paper has studied the behavior of an economy from the moment it opens up to trade until the moment it has "reached" its new long-run growth path. The framework of analysis was provided by a typical two-country model of innovation and growth with international financial capital flows. Transitional behavior was studied by performing a global-stability analysis. Global properties could be derived by exploiting interest-rate equalization due to perfect international capital flows and characteristics of the capital-market no-arbitrage condition. The crucial property of every path leading to the BGP turned out to consist in an international equalization of dividend rates which can be fruitfully applied for the analysis of related models as well. Given this property, long-run factor prices directly carry over to the entire saddle plane, provided that the productivity of labor relative to the other country does not change over time. This requires in the present framework perfect international knowledge spillovers. The innovation rate of the backward country overshoots its BGP value, whereas innovation rates of the leading economy approach the long-run innovation rate from below. The reason for convergence lies in perfect international knowledge spillovers and a frictionless shift of resources out of the production into the R&D sector during the adjustment period caused by a higher labor-per-firm endowment of the backward country. Cost advances in R&D do not play any role since factor prices equalized at every moment in time. An implication of international capital flows is a nonequalization of long-run levels of consumption indicating the consumption-smoothing aspect of international borrowing and lending. The initially rich country will always have a higher consumption level than the backward country. The accumulation of foreign wealth follows, in contrast to a model with directly productive capital, a smooth path which implies an initially positive and then negative trade balance of the leading country. Perfect international capital flows can even have the implication that innovation in the rich country comes to a halt for some time since all investments are concentrated in the backward country. Convergence is predicted also for this case. The final section has derived results for other models with perfect international knowledge spillovers but different technologies or preferences and has argued that various types of imperfect international dissemination of knowledge exclude convergence.

Appendix
The Balanced-growth Path

This appendix proofs two propositions. They are used for the description of the BGP in Sect. 3.1. The second is further of importance for concluding the proof of global stability at the end of Sect. 3.3.

Proposition 1: If countries innovate at the same rate, $\dot{n}^i/n^i = g$, sufficient conditions for the BGP to be an equilibrium are that firm values fall in both countries at that rate, $-\dot{v}/v = g$.

Proposition 2: If $\dot{n}^i/n^i = g$ and $-\dot{v}/v = g$, the world innovation rate is given by $g = (1-\alpha)L - \alpha\rho$.

Proof: By differentiating (6) with respect to time and by inserting expressions for profits and demand functions into the resulting equation and into the factor-market clearing condition (7), one obtains a simple system describing the model in terms of firm values (two co-state variables) and number of firms per country (two-state variables),

$$\frac{\dot{n}^i}{n^i} = \frac{n}{n^i}L^i - \alpha\frac{(v^i)^{-\varepsilon}}{n^{\mathrm{A}}(v^{\mathrm{A}})^{1-\varepsilon} + n^{\mathrm{B}}(v^{\mathrm{B}})^{1-\varepsilon}} \;, \tag{A.1}$$

$$\frac{\dot{v}^i}{v^i} = \rho - (1-\alpha)\frac{(v^i)^{-\varepsilon}}{n^{\mathrm{A}}(v^{\mathrm{A}})^{1-\varepsilon} + n^{\mathrm{B}}(v^{\mathrm{B}})^{1-\varepsilon}} \;. \tag{A.2}$$

Total expenditure as numeraire was set equal to unity which implies $\dot{E}^{\mathrm{A}}$

$+ \dot{E}^{\mathrm{B}} = 0$. By (1) this reads $(r - \rho)(E^{\mathrm{A}} + E^{\mathrm{B}}) = 0$ and consequently the nominal interest rate could be replaced by the time-preference rate, $r = \rho$.

Now insert $\dot{n}^i/n^i = g$ and $-\dot{v}^i/v^i = g$. This gives

$$g\frac{n^i}{n} = L^i - \alpha\frac{n^i}{n}\frac{(v^i)^{-\varepsilon}}{n^{\mathrm{A}}(v^{\mathrm{A}})^{1-\varepsilon} + n^{\mathrm{B}}(v^{\mathrm{B}})^{1-\varepsilon}} ,$$

$$-g = \rho - (1-\alpha)\frac{(v^i)^{-\varepsilon}}{n^{\mathrm{A}}(v^{\mathrm{A}})^{1-\varepsilon} + n^{\mathrm{B}}(v^{\mathrm{B}})^{1-\varepsilon}} .$$

Subtract the second equation from itself ($i = \mathrm{A, B}$) and find $v^{\mathrm{A}} = v^{\mathrm{B}} \equiv v$. Inserting this into the above equations and rearranging gives

$$g = L^i\frac{n}{n^i} - \frac{\alpha}{nv}, \quad g = \frac{1-\alpha}{nv} - \rho .$$

From the first equation we can readily deduce that on the BGP $n^{\mathrm{A}}/L^{\mathrm{A}} = n^{\mathrm{B}}/L^{\mathrm{B}} \iff n^{\mathrm{B}} = n^{\mathrm{A}}L^{\mathrm{B}}/L^{\mathrm{A}}$. Insert this equation into $(n^{\mathrm{A}}/n) + (n^{\mathrm{B}}/n) = 1$ and find $(n^{\mathrm{A}}/n)(1 + L^{\mathrm{B}}/L^{\mathrm{A}}) = 1$. This is equivalent to $n^{\mathrm{A}}/n = L^{\mathrm{A}}/L$ or, more generally,

$$\frac{n^i}{n} = \frac{L^i}{L} , \tag{A.3}$$

where $L = L^{\mathrm{A}} + L^{\mathrm{B}}$. Then, solving the first equation for nv gives $g = L - \theta/nv \iff \theta/nv = L - g \iff nv = \theta/(L-g)$ and inserting this result into the second gives an innovation rate of $g = [(1-\theta)/\theta](L-g) - \rho \iff g[1 + (1-\theta)/\theta] = [(1-\theta)/\theta]L - \rho \iff g = (1-\theta)L - \theta\rho$.

We summarize that imposing $\dot{n}^i/n^i - \dot{v}^i/v^i = g$ on the model as represented by (A.1) and (A.2) determines a solution of the model and hence an equilibrium. Initial conditions belonging to this solution were obtained in (A.3). This proofs Proposition 1. A consequence of the conditions $\dot{n}^i/n^i - \dot{v}^i/v^i = g$ is a constant innovation rate of $g = (1-\theta)L - \theta\rho$. This proofs Proposition 2. □

Further properties can now be easily derived. The value of all firms amounts to $nv = \alpha/(L-g) = \alpha/[L - (1-\alpha)L + \alpha\rho] = 1/(L+\rho)$, and the value of firms in country i is given by $n^i v = nv(n^i/n) = [1/(L+\rho)](L^i/L)$.

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Address of author: Klaus Wälde, Wirtschafts- und Sozialwissenschaftliche Fakultät, Graduiertenkolleg, Universität Dortmund, D-44221 Dortmund, Germany.