Economics Letters 44 (1994) 411–414

0165-1765/94/$07.00

On an additional condition for factor-price equalisation in intertemporal Heckscher–Ohlin models

Klaus Wälde*

Institut für Theoretische Volkwirtschaftslehre, University of Kiel, 24118 Kiel, Germany

Received 22 June 1993
Accepted 7 September 1993

Abstract

In order that factor-price equilisation results from free trade in intertemporal Heckscher–Ohlin models, individual discount rates must be equal. When this condition is violated and both countries lie within the cone of diversification, they produce both goods though factor prices are not equalised.

JEL classification: F12

1. Introduction

The pure theory of international trade based on Heckscher–Ohlin models distinguishes between two possible outcomes of international trade: either factor prices are equalised and both countries are incompletely specialised or at least one country is completely specialised and no factor-price equalisation obtains. The assumptions necessary for factor-price equalisation (FPE) are free trade, absence of transportation costs, that both countries produce with the same technology and the absence of factor-intensity reversal. Taking into consideration models combining the Heckscher–Ohlin approach with endogenous growth [see, for example, Grossman and Helpman (1991, ch.7), Dinopoulos et al. (1993)] one becomes aware of the fact that in an intertemporal context, another condition adds to the one mentioned above: individual discount rates must not differ between countries.

In what follows this effect is demonstrated in the, what we believe, simplest possible way by using a slightly modified version of the Grossman and Helpman (1989) model. The modification consists in fixing the number of factors to two and in assuming fixed R&D costs [see, for example, Barro and Sala-i-Martin (1992)]. Note that the effect presented does not depend on these simplifications but arises in models with positive long-run growth rates and more sophisticated R&D specifications too [Wälde (1993)].

In section 2 the two-country model will be briefly presented. Section 3 will prove the main proposition before final remarks conclude the paper.

* Financial support by the state of Schleswig-Holstein is gratefully acknowledged.

SSDI 0165-1765(93)00347-Q

2. A two-country world

The consumer side is modelled by using the concept of a representative household which maximises utility over an infinite horizon. The utility function is given by

$$U^i(t) = \int_t^\infty \mathrm{e}^{-\rho^i[\tau - t]} \left(\frac{\sigma}{\theta} \log \int_0^n x_c(j)^\theta \, \mathrm{d}j + (1-\sigma) \log Z_c \right) \mathrm{d}\tau \,, \quad 0 < \theta, \sigma < 1 \,, \tag{1}$$

where ρ^i is the individual discount rate of consumers (with $i = \mathrm{A}, \mathrm{B}$ indicating the country) and $n = n^\mathrm{A} + n^\mathrm{B}$ is the measure for the number of varieties available for consumption. With these preferences both representative consumers (one in each country) devote a fraction σ of their spending to the differentiated good and the remaining fraction $1 - \sigma$ to the homogeneous good. This allows us first to derive static demand functions, insert them into the utility function (1) and then maximise it with respect to expenditure subject to an intertemporal budget constraint to derive an optimal allocation of expenditure over time. We further know that due to identical production functions and the equalisation of factor rewards between *sectors*, all varieties produced in *one* country are equally priced. All of this [1] gives aggregate demand functions for the differentiated good of the form [see, for example, Helpman and Krugman (1985, ch. 6)]

$$x^i = \frac{(p_x^i)^{-\epsilon}}{n^\mathrm{A}(p_x^\mathrm{A})^{1-\epsilon} + n^\mathrm{B}(p_x^\mathrm{B})^{1-\epsilon}} \sigma(E^\mathrm{A} + E^\mathrm{B}) \,, \tag{2}$$

and for the homogeneous good

$$Z^\mathrm{A} + Z^\mathrm{B} = \frac{1-\sigma}{p_Z} (E^\mathrm{A} + E^\mathrm{B}) \,. \tag{3}$$

The intertemporal allocation of expenditure follows [see, for example, Grossman and Helpman (1989)]:

$$\frac{\dot{E}^i}{E^i} = r^i - \rho^i \,. \tag{4}$$

Both countries are endowed with labour and human capital and consist of two manufacturing sectors, each one employing both factors for producing one homogeneous and one differentiated final good. The sectors can be ranked by their factor intensity, with the sector producing the differentiated good being the most human capital intensive one. Perfect competition in the food industry implies

$$p_Z = c_Z(w_\mathrm{L}^i, w_\mathrm{H}^i) \,, \tag{5}$$

where $c_Z(w_\mathrm{L}^i, w_\mathrm{H}^i)$ are unit cost functions whereas manufacturers of the differentiated products practice mark-up pricing:

$$p_x^i = \frac{1}{\theta} c_x(w_\mathrm{L}^i, w_\mathrm{H}^i) \,. \tag{6}$$

Prior to producing a differentiated good a new variety has to be developed which requires fixed

[1] Details are available from the author upon request.

costs ϕ which do not differ between countries. Allowing for free entry and exit, an equilibrium will be characterised by an equalisation of R&D costs and the discounted sum of future profits:

$$\phi = \int_{t}^{\infty} e^{-[R(\tau)-R(t)]}\pi^{i}(\tau)\,d\tau\,. \tag{7}$$

The discount factor is given by $R(s) = \int_0^s r(u)\,du$, where $r(u)$ is the interest rate.

Finally we have to specify factor market clearing conditions. The supply of factors is assumed to be totally inelastic, the unit demand functions for factors result from cost minimisation and the application of Sheppard's lemma. With the assumption that factor rewards adjust such that markets clear at every moment in time, we obtain

$$\begin{aligned} a_{LX}(w_L^i, w_H^i)n^i x^i + a_{LZ}(w_L^i, w_H^i)Z^i &= L^i\,, \\ a_{HX}(w_L^i, w_H^i)n^i x^i + a_{HZ}(w_L^i, w_H^i)Z^i &= H^i\,. \end{aligned} \tag{8}$$

Thus the model turns out to be identical to the Helpman and Krugman (1985, ch. 7) model where the static zero-profit condition is replaced by an intertemporal one and the utility maximisation calculation of the consumers is enlarged by an intertemporal component. The model can be solved for equilibrium prices, output and the number of varieties by using Eqs. (2), (3), (5)–(7) and (8) with total expenditure $E^A + E^B$ chosen as numeraire.

3. Unequal factor rewards and incomplete specialisation

Differentiating (7) with respect to time yields the usual capital market no-arbitrage conditions which express the indifference of agents between a riskless loan or buying a share of a firm:

$$\pi^i = r^i\phi\,. \tag{9}$$

With Eqs. (6) and (2) current profits are

$$\pi^i = (p_x^i)^{1-\epsilon}\frac{(1-\theta)\sigma(E^A+E^B)}{n^A(p_x^A)^{1-\epsilon} + n^B(p_x^B)^{1-\epsilon}}\,.$$

By subtracting Eq. (9) from itself ($i =$ A,B) we find

$$((p_x^A)^{1-\epsilon} - (p_x^B)^{1-\epsilon})\frac{(1-\theta)\sigma(E^A+E^B)}{n^A(p_x^A)^{1-\epsilon} + n^B(p_x^B)^{1-\epsilon}} = (r^A - r^B)\phi\,. \tag{10}$$

We know that factor-price equalisation requires equalisation of goods prices due to the one-to-one mapping between goods prices and factor rewards given by the pricing equations (5) and (6) under the assumption of no factor-intensity reversal. Thus if prices for the differentiated goods differ, factor prices cannot be equalised. Equation (10), however, tells us that prices for the differentiated good are equalised only if interest rates are equalised. In an equilibrium, however, expenditure of every country is constant and thus via (4) the interest rate has to equal the individual discount rate. Now if they differ, goods prices for the differentiated good differ and consequently factor prices do not equalise.

What remains to be shown is that both countries are incompletely specialised in the long-run equilibrium. This can be easily seen from the factor market clearing conditions: rewrite them as

$L_X^i + L_Z^i = L^i$ and $h_X^i L_X^i + h_Z^i L_Z^i = H^i$, where h_X^i and h_Z^i are the human capital intensities in sectors X and Z, respectively. Solving for L_X^i and L_Z^i gives the usual conditions for incomplete specialisation: if the factor endowment of country i lies within a certain cone of diversification whose boundaries are given by $h_Z^i L^i < H^i < h_X^i L^i$, then both L_X^i and L_Z^i are positive. Consequently, H_X^i and H_Z^i are positive, too, and country i is incompletely specialised. Therefore, as long as individual discount rates differ and both countries lie within the cone of diversification, both countries produce both goods, but factor prices differ.

4. Conclusion

In static free trade models with two goods and two factors, FPE results if both countries are not too dissimilarly endowed, factor-intensity reversal is excluded, production takes place with identical technologies and transportation costs are neglected. Dynamic models further require identical individual discount rates. If discount rates differ, factor prices cannot be equalised as long as both countries produce both goods.

Finally, we add that, since unequal discount rates imply unequal interest rates, we obtain an equilibrium only under the assumption of internationally immobile capital. This does not present a problem as long as we make this assumption or we exclude physical capital from the model (as is often done in models of endogenous growth). In an extension of the above model with physical capital as an additional factor, this should be taken into consideration.

References

Barro R.J. and X. Sala-i-Martin, 1992, Public finance in models of economic growth, Review of Economic Studies 59, 645–661.

Dinopoulos, E., J.F. Oehmke and P.S. Segerstrom, 1993, High-technology-industry trade and investment, Journal of International Economics 34, 49–71.

Grossman, G.M. and E. Helpman, 1989, Product development and international trade, Journal of Political Economy 97, 1261–1283.

Grossman, G.M. and E. Helpman, 1991, Innovation and growth in the global economy (MIT Press, Cambridge, MA).

Helpman, E. and P. Krugman, 1985, Market structure and foreign trade (MIT Press, Cambridge, MA).

Wälde, K., 1993, Unequal factor rewards and incomplete specialisation in a Heckscher–Ohlin model of endogenous growth, mimeo, Universität Kiel (to be presented at the Econometric Society European Meeting, 1993).