Review of International Economics, 15(2), 414–429, 2007

International Trade, Hedging, and the Demand for Forward Contracts*

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Abstract

One of the main results of the literature on the effects of uncertainty on trade states that uncertainty should not matter in the presence of well-developed forward markets. Empirical studies, however, do not support this result. We derive the demand for forward cover in a small open economy with terms-of-trade uncertainty. Adopting a standard and more realistic decision structure than the one usually used in this literature, we find that risk-averse agents will not buy forwards at an unbiased price. Agents treat forward contracts as an asset rather than as an insurance. This is the reason why, when calibrating the model, only 17% of imports are covered by forwards.

1. Introduction

International trade in goods is characterized by uncertainty. Common sense and economic theory suggest that exporters, importers, and households should try to hedge against this uncertainty. Natural candidates for hedging instruments are future and forward contracts. In fact, Ethier (1973) introduced the separation theorem and the full-hedge theorem under exchange rate uncertainty, showing that demand for forward contracts perfectly compensates uncertainty. Benninga et al. (1985) and Kawai and Zilcha (1986) additionally discussed price-level uncertainty, obtaining the same results. Recently, this strong result has been subject to some qualifications. Viaene and Zilcha (1998), for example, consider additionally output and cost uncertainty and find that under this set-up full-double hedge and separation fail to hold. Adam-Müller (2000) introduces inflation risk which cannot be hedged away and finds that full-hedge and separation break down if the two sources of risk in the model are not statistically independent. Market structure issues have been addressed as well, examples being Eldor and Zilcha (1987) and Broll and Zilcha (1992).

The empirical literature, though sparse, does not support the strong theoretical predictions of the early literature. As Carse et al. (1980) and others have shown, roughly only one-third of the value of international trade is covered by forward contracts. Even equity flows are only poorly hedged. According to Hau and Rey (2003), only 8% of US equity holdings abroad are hedged against exchange rate risks. Furthermore, there exists a lively debate in the empirical literature as to whether exchange rate volatility depresses trade levels or not. This debate is related to the issue of demand for forwards in that often the argument is made that as long as agents have access to well-developed forward markets, the uncertainty should not matter. Strikingly, the

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evidence is rather mixed and seems to be independent of the existence of well-developed forward markets. A survey on the empirical evidence is provided by Coté (1994), and Wei (1998) discusses the underlying causes.

This paper reconciles empirical findings with theoretical considerations. We show that by allowing agents to optimally choose their consumption bundle after the resolution of price uncertainty—which is in contrast to the literature on forwards but standard in, for example, macro models with uncertainty—forward contracts resemble normal uncertain assets rather than insurance contracts. As forward contracts tend to have lower returns than, for example, physical capital, agents do not hold many of those assets and trade flows are only poorly hedged.

We build an infinite horizon small open-economy model where one good is domestically produced with capital and labor, and another good is imported. Both goods are consumed. Capital is accumulated and risk-averse households hedge optimally against terms-of-trade uncertainty.[1] One forward contract allows (and obliges) them to buy one import good in the next period at a fixed price $\bar{p}^Y$.

We first study the determinants of demand for forwards. We show that the exogenous internationally given forward price $\bar{p}^Y$ is the crucial determinant of demand for forwards. When this forward price equals the expected price of the import good, i.e. when the forward price is unbiased, risk-averse households do not want to buy any forwards—they would actually want to sell forwards. When the forward price equals the price at which risk-neutral households would be indifferent, risk-averse households demand a positive amount of forward contracts.

Risk-averse households want to sell forwards at unbiased prices as their utility function is concave in consumption levels. With consumption levels optimally chosen *ex post*, indirect utility functions of individuals exhibit convexity in prices, though still concavity in expenditure. As expenditure is a function of prices as well, overall, the indirect utility function exhibits convexity in prices and households are actually (price-) risk lovers. Positive demand therefore requires a price that is sufficiently low: for example, the price offered by risk-neutral households. Intuitively, we could think of the risk-averse households as not willing to commit themselves to a consumption decision when faced with price uncertainty. They do not want to give away the option to adjust their consumption bundles.

We then calibrate the model by using realistic and reasonable parameter values. We find that between 10% and 20% of international trade is covered by forward contracts. The low ratios cited in the empirical literature are therefore not surprising and may reflect the curvature of utility functions of utility-maximizing households. Partial-equilibrium set-ups, or set-ups focusing on risk-neutral firms, should therefore be extended to take this aspect into consideration.

We are not the first to find that full-hedge theorem and separation theorem do not hold. As argued above, there is a substantial literature that finds that these two theorems will not hold as soon as certain conditions are violated. Our result, however, is derived in a completely different manner. The crucial point is the decision structure of our agents. The standard approach assumes that *all* decisions are made *before* the resolution of uncertainty. In contrast, we employ an alternative decision rule, which is commonly used in macro models with uncertainty. In the first period, still before resolution of uncertainty, the agents decide on their level of hedging and in the second, *after* the uncertainty is resolved, the agents actually make their consumption decision. Following this approach, agents will never be able to eliminate uncertainty from their budgets and hence are faced with a tradeoff. As a consequence, risk-averse agents will never buy forward cover under unbiased insurance prices.

2. The Model

Technologies

We study a small open economy that produces one good X that is internationally traded. It imports a foreign consumption good Y which is not domestically produced. Domestic production requires capital K and labor L, which are nontradable:

$$X_t = X(K_t, L_t). \tag{1}$$

Time is discrete and variables are indexed by t. The production function $X(\cdot)$ has the standard neoclassical properties. Firms produce under perfect competition and factor rewards w_t^L and w_t^X for labor and capital are given by their value marginal productivities:

$$w_t^L = p_t^X \partial X_t / \partial L_t, \quad w_t^K = p_t^X \partial X_t / \partial K_t. \tag{2}$$

The number of units of the import good to be exchanged for one unit of the export good, i.e. international terms of trade p_t^X/p_t^Y at a point in time t, are exogenously given to the economy and random. Before any trade in t takes place, prices for period t become common knowledge. Prices for period $t+1$ are not known in t but the density function $f(p_\tau^X/p_\tau^Y)$ of p_τ^X/p_τ^Y for $\tau > t$ is common knowledge. In what follows, we choose X as *numéraire* and denote its price by p^X:

$$p_{t+1}^X = p_t^X \equiv p^X.$$

One can therefore think of the price of the domestic good as a deterministic price and of the price of the foreign good as stochastic.

Domestic output X from the production process (1) is used for domestic consumption C_t^X, exports X_t^E, and gross investment I_t:

$$X_t = C_t^X + X_t^E + I_t. \tag{3}$$

Letting δ capture depreciation, capital grows according to:

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{4}$$

In addition to producing the good Y, foreign agents offer forward contracts. At transaction costs of $\chi \geq 0$ per unit to be paid in t, domestic agents can buy forward contracts from foreign agents. Thus, foreign agents agree in t to sell in $t+1$ one unit of the foreign good at the exogenous internationally given price $\bar{p}^Y$. This is equivalent to fixing in t next period's terms of trade at $p^X/\bar{p}^Y$. When forward contracts of total volume D_t are signed, foreign agents agree to sell D_t units of good Y at $\bar{p}^Y$ in $t+1$. Domestic buyers commit to buy in $t+1$ at this price, irrespective of the realization of p_{t+1}^Y.[2]

Households

The horizon of the economy is infinite. Agents in this economy live for two periods. They work in the first period of their life and consume in the second period. Consumption in the second period comprises both the domestically produced good and the foreign good.

Preferences and budget constraints The utility function of households is given by

$$v = v(u(C_X, C_Y)),$$

where $u(C_X, C_Y)$ is some homothetic utility function and $v(\cdot)$determines the degree of risk aversion. For illustrating purposes, we will later use

$$u(C_X, C_Y) = C_X^{\alpha} C_Y^{1-\alpha}, \quad 0 < \alpha < 1, \tag{5}$$

$$v(x) = \frac{x^{\sigma}}{\sigma}, \quad 1 \geq \sigma > 0. \tag{6}$$

Note that the utility function (5) displays risk aversion towards the consumption *levels*. Risk aversion in total consumption *expenditure* is given for $0 < \sigma < 1$; risk neutrality in consumption expenditure would be represented by $\sigma = 1$.

A household's first-period budget constraint equates labor income with savings and transaction costs for financial contracts D_t:[3]

$$w_t = s_t + \chi D_t. \tag{7}$$

Savings are used to buy capital goods s_t/p^X. There is the implicit assumption of a market in which today's old, being the owners of the capital stock, sell it to today's young in exchange for consumption good X, which in turn constitutes the wage of today's young. The sum over all individual savings equals the current capital stock (i.e. after depreciation) plus additional aggregate investment:

$$K_{t+1} = (1-\delta)K_t + I_t = \frac{s_t}{p^X} L. \tag{8}$$

In the second period, households use all of their wealth and other income for financing consumption expenditure e_{t+1}. End-of-second-period wealth amounts to $p^X(1-\delta)(s_t/p^X) = (1-\delta)K_{t+1}$. Factor rewards for wealth amount to $p^X(\partial X_{t+1}/\partial K_{t+1})(s_t/p^X)$. Income from forward contracts is $(p_{t+1}^Y - \bar{p}^Y)D_t$, which might be negative. Hence

$$e_{t+1} \equiv p^X C^X + p_{t+1}^Y C_Y = (1 + r_{t+1})p^X \frac{w_t - \chi D_t}{p^X} + (p_{t+1}^Y - \bar{p}^Y)D_t, \tag{9}$$

where we defined

$$1 + r_{t+1} \equiv 1 + \frac{\partial X_{t+1}}{\partial K_{t+1}} - \delta \tag{10}$$

and savings s_t were replaced by using the first-period budget constraint (7).

The second-period budget constraint (9) shows nicely that payoffs $(p_{t+1}^Y - \bar{p}^Y)D_t$ from forward contracts are positive and therefore a second-period source of income when the price p_{t+1}^Y of good Y is sufficiently high relative to its exogenous price $\bar{p}^Y$ specified one period before. Forward contracts imply a loss in the case of low price of good Y. Of course, bad terms-of-trade shocks leading to income and good terms-of-trade shocks leading to losses from forward contracts are the reason why forwards exist: they insure against terms-of-trade shocks.

This budget constraint also shows that households can not insure fully against terms of trade risk. Forward contracts refer to a certain amount of goods that can be purchased at this fixed price $\bar{p}^Y$. As the actual amount of goods consumed depends on the realization p_{t+1}^Y of the price, some uncertainty always remains. This is the crucial departure of our model from the classic set-ups in the hedging literature—see Ethier

(1973, p. 496) and Benninga et al. (1985, p. 540). There, firms decide today in t how much they will produce tomorrow in $t+1$. This allows them to fully insure against uncertainty in the price of their output good. The well-known separation theorem of no uncertainty after hedging results. If our agents knew how much they will consume tomorrow, full hedging would be possible as well. They will never know, however, as price uncertainty also has an income effect.

A no-bankruptcy constraint In order to avoid insolvency of households, we have to introduce a no-bankruptcy constraint. Our point of departure is the expenditure equation (9). As negative expenditure is not feasible, we argue that the worst that can happen to the budget of our agents is an expenditure of zero:

$$e_t = (1+r_{t+1})w_t + \left(p_{t+1}^Y - (1+r_{t+1})\chi - \bar{p}^Y\right)D_t = 0.$$

Solving for D_t yields

$$D_t = \frac{(1+r_{t+1})w_t}{(1+r_{t+1})\chi + \bar{p}^Y - p_{t+1}^Y}.$$

Regarding our forward, the most unfavorable situation for households is $p_{t+1}^Y = 0$. Prudence thus demands that the amount of D_t an agent is allowed to purchase shall never be any greater than

$$D_t \leq \frac{(1+r_{t+1})w_t}{(1+r_{t+1})\chi + \bar{p}^Y}. \tag{11}$$

This condition intuitively makes sense: the greater the contracted price $\bar{p}^Y$ and the greater the costs of forward cover χ, the smaller the number of forwards the agents are allowed to buy. Similarly, the greater the interest rate and wage income w_t, the greater the amount of D_t the agents can commit to. Note that the interest rate r_{t+1} is deterministic and hence known in period t, since the capital stock is deterministic and there are no technology shocks in the model. The expression $(1 + r_{t+1})w_t$ then simply gives maximum period $t+1$ income, computed in period t. The denominator of (11) in turn gives the highest possible costs of the forward position, evaluated in period t. This ratio gives the number of forwards D_t an agent can buy such that in the most unfavorable realization of forward prices the agent still has a non-negative expenditure level.

3. Solving the Model

The Maximization Problem of Households

The maximization problem of households consists in choosing the amount D_t of forward contracts and optimal consumption levels C_X and C_Y such that expected utility $\mathrm{E}[v(u(C_X, C_Y))]$ is maximized, given the budget constraint (9).

Conceptually, maximization can be subdivided into two steps. The second step consists in allocating consumption expenditure to goods X and Y, taking consumption expenditure as given. This second subproblem is solved after realization of terms of trade. It is therefore a choice under certainty. The Cobb–Douglas specification (5) implies

$$C_{t+1}^X = \frac{\alpha e_{t+1}}{p^X}, \tag{12}$$

$$C_{t+1}^Y = \frac{(1-\alpha)e_{t+1}}{p_{t+1}^Y}. \tag{13}$$

These equations hold at each point in time and determine consumption levels after uncertainty has been resolved.

The first step consists in choosing the optimal amount D_t of forward contracts that maximizes $\mathrm{E}[v(e_{t+1}/P(p^X, p_{t+1}^Y))]$, where $v(e_{t+1}/P(p^X, p_{t+1}^Y))$ is indirect utility where consumption levels in the homothetic utility function $u(C_X, C_Y)$ have been replaced by optimal consumption levels. Utility $u(C_X, C_Y)$ can then be written as expenditure divided by the price index. In the Cobb–Douglas case, the price index reads $P(p^X, p_{t+1}^Y) = \Phi p_X^\alpha p_Y^{1-\alpha}$, where Φ is a constant. Expenditure is given by (9).

This two-step solution to our maximization problem is made possible by assuming that consumption takes place only when agents are old. If consumption were to take place in both periods, the consumption choice in the first period would be linked to the saving decision. The system that would have to be analyzed would be more complicated (as an intertemporal consumption rule would have to be added).

The solution to this problem is then given by the first-order condition:

$$\mathrm{E}\left[v'\left(\frac{e_{t+1}}{P(p^X, p_{t+1}^Y)}\right)\frac{p_{t+1}^Y - (1+r_{t+1})\chi - \bar{p}^Y}{P(p^X, p_{t+1}^Y)}\right] = 0, \tag{14}$$

where the expectations operator refers to the entire bracket $[\cdot]$. This condition consists of two parts. The first is marginal utility $v'(\cdot)$, here expressed in the form of the indirect utility function. Marginal utility is positive but decreasing in consumption levels, or, as stated here, increasing in expenditure and decreasing in prices. The denominator of the second term, $p_{t+1}^Y - (1 + r_{t+1})\chi - \bar{p}^Y$, represents the realized nominal return from the forwards. Its expected value is negative under unbiased (or actuarially fair[4]) forwards, i.e. if $\mathrm{E}[p_{t+1}^Y] = \bar{p}^Y$, since the term $(1 + r_{t+1})\chi$ representing the opportunity costs of entering the forward market enters negatively. If forwards could be obtained without costs, clearly these opportunity costs would vanish and unbiased forwards would have an expected nominal return of zero. Dividing the nominal return by the price index gives the complete second term, the real return of the forward contract.

Now that the meaning of the two components of (14) is clear, the intuition of this first-order condition is easier to see. The expectations operator is an integral in our case where terms of trade is a continuous random variable. This integral can be split into negative and positive parts. As long as $p_{t+1}^Y < (1 + r_{t+1})\chi + \bar{p}^Y$, the realized price is in the "loss region." Marginal utility is multiplied by a negative number and this interval of the integral contributes negatively. Concavity of the utility function with respect to the amount of forwards implies that as long as (14) is negative, the agents have too many forwards and hence should decrease holdings. On the other hand, as soon as $p_{t+1}^Y > (1 + r_{t+1})\chi - \bar{p}^Y$—the "win region"—marginal utility contributes positively. Again, concavity tells us that as long as (14) is positive agents should increase holdings of D_t. However, given positive costs to obtain forward cover, i.e. $\chi > 0$, increasing D_t will increase r_t, hence opportunity costs will rise as well, up to a point where marginal utility will fall in D_t. Hence the optimal amount of D_t is such that the positive and the negative components of the integral just balance.

Reduced Form

The reduced form of the model consists of two equations. The capital stock in the next period is given by savings today times the number L of individuals and divided by the price of one unit of capital and is given by (8). With the first-period budget constraint (7) giving individual savings, we obtain

$$K_{t+1} = \frac{p_t^X \partial X_t / \partial L - \chi D_t}{p^X} L, \tag{15}$$

where the wage rate was replaced by its value marginal product (2).

The amount of forward contracts is determined by the first-order condition (14). Consumption of the old is given by the current capital stock, interest payments on the current capital stock plus income (or losses) from forward contracts. Using the budget constraint (9), where wages w_t were replaced by value marginal productivities in (2), expenditure in (14) therefore equals

$$e_{t+1} = (1 + r_{t+1}) p^X \partial X(K_t, L) / \partial L + \left(p_{t+1}^Y - (1 + r_{t+1})\chi - \bar{p}^Y\right) D_t. \tag{16}$$

Equilibrium in our economy is therefore described by equations (14) and (15), given (16). These equations determine the two variables K_t and D_t, given an initial capital stock K_0.

Equation (15), determining the evolution of capital, shows that next period's capital is known in t. By contrast, expenditure (16) is uncertain when some forward contracts are signed. This makes consumption levels of both goods and exports and imports uncertain. If no forward contracts are signed ($D = 0$), expenditure is deterministic, consumption of good X would be deterministic but consumption of good Y would be stochastic.

Steady State

In the steady state, the capital stock is the same in each period. Variables that are constant are printed without a time subscript; all stochastic variables are denoted by a tilde (˜). The capital stock is then determined by

$$K = \frac{p^X \partial X / \partial L - \chi D}{p^X} L \tag{17}$$

and is therefore a deterministic variable. Domestic production (1) is then deterministic as well, $X = F(K, L)$. Steady-state expenditure $\tilde{e}$ is given from (16) as

$$\tilde{e} = (1 + r) p^X \partial X / \partial L + \left(\tilde{p}^Y - (1 + r)\chi - \bar{p}^Y\right) D \tag{18}$$

and remains stochastic. Using (18), D follows implicitly from the first-order condition (14):

$$\mathrm{E}\left[v'\left(\frac{\tilde{e}}{P(p^X, \tilde{p}^Y)} \right) \frac{\tilde{p}^Y - (1 + r)\chi - \bar{p}^Y}{P(p^X, \tilde{p}^Y)} \right] = 0. \tag{19}$$

4. Equilibrium Properties

Given the steady-state quantities of the capital stock K and forward contracts D as determined in (17) and (19) with (18), will agents want to hold a positive amount of forwards? This will be analyzed in the next subsection. In order to obtain an idea about quantitative predictions, we calibrate the model in the subsequent section and provide numerical results afterwards. We also perform a comparative static analysis and finally introduce options as an alternative to forwards. By deriving several equilibrium properties under options, the properties of forwards will also become clearer.

The Equilibrium Demand for Forwards

We now present three important results with respect to the existence of interior solutions, i.e. a positive demand for forwards D in the steady state. For simplicity, we set transaction cost equal to zero, $\chi = 0$, in what follows. Note that this implies by (17) a capital stock that is independent of the choice of D. All proofs will be found at the website address given in the Appendix.

THEOREM 1. *Risk-averse agents will not buy forward cover at unbiased prices, i.e.* $E[\tilde{p}^Y] = \tilde{p}^Y$.

We illustrate this result in Figure 1. It plots expected utility of agents in the steady state, $E[v(\tilde{e}/P(p^X, p^Y))]$, as a function of forwards D, taking expenditure from (18) into account.[5] The figure shows how expected utility of households depends only on forwards, provided that they anticipate the choice of consumption levels, and thereby illustrates the maximization problem of section 3. Since our objective function is globally concave in D (see the Appendix), the sign of the first derivative of this function with respect to D at point $D = 0$ determines whether or not there is an interior solution. As plotted above, expected utility would be maximized at a negative D. Agents therefore do not want to hold forward contracts.

In light of the existing literature on the topic this result is rather surprising. The standard result states[6] that if an unbiased forward market exists, agents use this market to avoid all uncertainty, i.e. they obtain full cover for their position. The crucial difference of our model to the literature lies in the timing structure. The main body[7] of the literature assumes that all decisions are made *before* uncertainty is resolved. In contrast, we assume, as is standard in (for example) stochastic macro models, that although the agents decide on the optimal amount of forward cover before uncertainty is resolved, their consumption decision is made after the resolution of the price uncertainty. Under

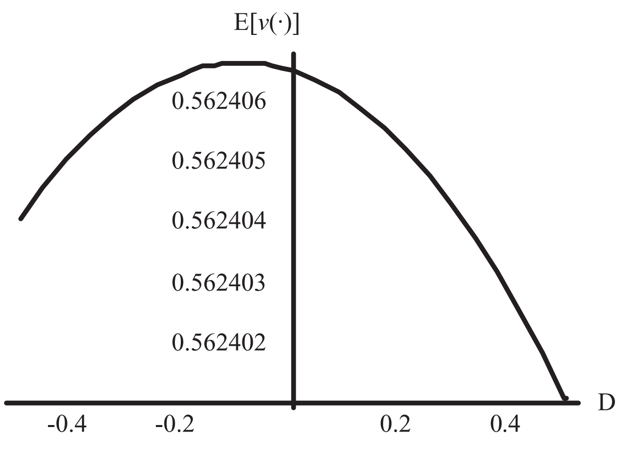

Figure 1. Expected Utility as a Function of Forwards D

this set-up, buying forward contracts amounts to no less than restricting one's possibilities to adjust to price realizations. Risk-averse agents will not give away this opportunity. Put differently, as agents cannot fully insure against terms-of-trade risk (see the discussion after (10)), a forward contract is basically a risky asset and the separation theorem fails. It is clear that there are some decisions that will be made in advance and for this part the analysis of the existing literature would be appropriate. We believe, however, that most consumption decisions are made when actual consumption takes place and prices are known.

Theorem 2. *Risk-averse agents will only buy forward cover for sufficiently low* $\bar{p}^Y$*, i.e.* $\mathrm{E}[\tilde{p}^Y] > \bar{p}^Y$.

Note that this result follows from the first theorem. One possible interpretation would be that if $\bar{p}^Y$ is lower than the expected value of the price p_{t+1}^Y, the average return of a forward position will be positive. Thus the agents will be compensated for giving up their possibility to adjust their consumption bundle according to the price realizations in the next period. Hence the agents are willing to hold a forward position.

Theorem 3. *If the exogenous forward price amounts to* $\bar{p}^Y = \mathrm{E}[p_Y^{\alpha}]/\mathrm{E}[p_Y^{\alpha-1}]$*, i.e. the price risk-neutral households would offer, risk-averse agents will buy forward contracts.*[8]

To illustrate the third result, imagine a graph similar to Figure 1 for risk-neutral households. Letting the forward price be given by the risk-neutral price $\bar{p}^Y = \mathrm{E}[p_Y^{\alpha}]/\mathrm{E}[p_Y^{\alpha-1}]$, the slope of the expected utility at $D = 0$ is zero. The slope of expected indirect utility at this point $D = 0$ can be expressed, for any given value of $\bar{p}^Y$, as a function of the degree of risk aversion. Theorem 3 essentially states that the more risk averse agents are, the larger the slope becomes. Hence, moving from risk neutrality, i.e. $\sigma = 1$, to risk aversion is equivalent to shifting the whole graph to the right. This in turn implies that the forward price $\bar{p}^Y$ at which the risk-neutral agents are just indifferent between buying and selling induces a positive demand by any risk-averse agent.

Note that these results may be somewhat surprising, given the "full-hedge theorem"—as in Ethier (1973) or Kawai and Zilcha (1986)—we normally encounter in the literature. The reason for this is that our model differs from the usual models such that agents always face uncertainty through the price-index channel, whereas in the former models there is the possibility to avoid all uncertainty, for agents completely decide upon their plans in period one. Risk-averse agents do not want to lose the ability to adjust to price shocks in the next period, whereas risk-neutral agents are indifferent towards this opportunity.

Secondly, we have another factor at work here. By buying forward contracts the agents trade one risk against the other. Holding a forward position means that risk now directly affects nominal income. This can be easily seen from (9). Risk aversion regarding nominal income and the uncertainty through the price-index channel are the reasons for the agents asking for more than unbiased forwards.

The convexity of the indirect utility function with respect to the prices is illustrated in Figure 2. It shows indirect utility as a function of the foreign price. Convexity of indirect utility with respect to prices implies that agents prefer any linear combination of prices to the average of this linear combination. Hence, agents are in fact risk-lovers with respect to period two price uncertainty. Note that this result hinges entirely on the timing assumption of the consumption decision.

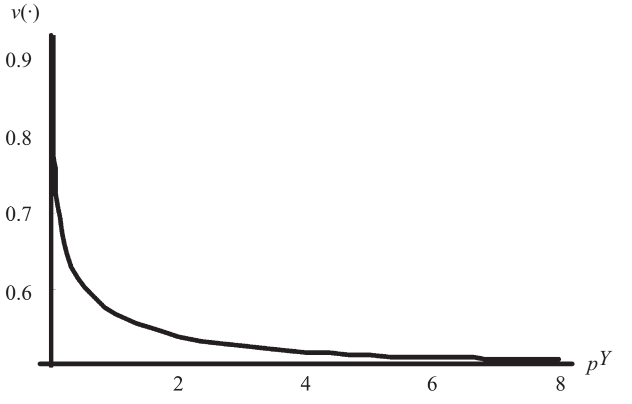

Figure 2. The Indirect Utility Function which is Convex in p^Y

Table 1. Parameter Values Used for Calibrating

Parameter	L	α	β	δ	χ	σ	Φ	S	p_X
Value	100	$\frac{4}{5}$	$\frac{3}{10}$	0.54	$\frac{1}{100}$	$\frac{1}{2}$	$\frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$	1	1

Calibrating the Model

We will now calibrate our model as this allows us to provide quantitative results in the next subsection and to subsequently perform a comparative static analysis (see Table 1). We begin by discussing the chosen values. Solving the model numerically involves computing values of both D and K, which satisfy (17) and simultaneously (19). To get numerical results, we need to specify a couple of parameters and the underlying distribution. As far as possible, this is achieved by drawing on real-world data.

As a first step, we specify the production technology by a Cobb–Douglas form, $X(K, L) = SK^{\beta}L^{1-\beta}$. The scale parameter for the technology, S, is set to one. Equation (17) depends on various parameters: P_X is the price of the *numéraire* good and can thus be set to one. Transaction costs for forwards are captured by χ in our set-up. One could think of χ as including some kind of market price of the forward contract or of obtaining forward cover. These two concepts are in fact quite different. In reality, the market price of the forward cover is quite small, whereas the real costs of obtaining forward cover may very well be substantial.[9] This leaves some room for determining the value of χ and thus we will set this value arbitrarily, but close to zero. In our calibration we use 1/100. The size of the population, as the total factor productivity (TFP) measure S, is just a scale parameter and therefore no further elaboration is necessary. We set $L = 100$. The output elasticity β in our production function reflects relative shares of capital and labor and is commonly found (e.g. in Maddison, 1987, p. 658), to be around 0.3.

The second reduced-form equation (19) and (18) require the specification of some parameters as well. Depreciation is assumed to be 2.5% per year. With one period representing 30 years in our two-period OLG set-up, we have $\delta = 0.54$.[10] The price $\overline{p}^Y$ is

Table 2. Parameters of the Log-normal and Underlying Normal Distribution

Distribution	$\mathrm{E}[\tilde{p}^Y]$	$\mathrm{Var}[\tilde{p}^Y]^2$
Log-normal	0.1149	0.0071
Underlying normal	1.1261	0.0103

determined by the price at which risk-neutral individuals would offer the forwards, i.e.[11]

$$\bar{p}^Y = \frac{\mathrm{E}\left[\tilde{p}_Y^{\alpha}\right] - (1+r)\chi \mathrm{E}\left[\tilde{p}_Y^{\alpha-1}\right]}{\mathrm{E}\left[\tilde{p}_Y^{\alpha-1}\right]}. \tag{20}$$

Equation (20) is determined by using the first-order condition (19), setting $\sigma = 1$ and solving for $\bar{p}^Y$ (see the Appendix). The parameter of the utility function, α, determines the share of domestic in total consumption. Using data from *Statistisches Bundesamt*, the empirically observed share of foreign products in total consumption in Germany is approximately 0.8. To determine the most appropriate distribution, we obtained monthly price index data for both import prices and export prices over the period January 1962 until January 2002, leaving us with 482 observations. Dividing the import index by the export index amounts, in terms of our model, to obtaining the price series p_t^Y. The shape of the histogram suggested choosing a log-normal distribution, which is an assumption commonly made, for example, in the finance literature.[12] The parameters of the distribution were obtained by maximum-likelihood estimation.[13] The estimates are shown in Table 2.

A Numerical Solution

We now present a simulation result for a small country. Under log-normal distributed price uncertainty, using the parameter specification we presented above, we found that the economy will buy a total amount of 1.62 units of forward contracts, given the price risk-neutral agents would offer. The capital stock and thus GDP of the economy can be calculated and, using the mean on the distribution as the realization of the price in period two, the economy will import 9.5 units of good *Y*. This means that the forward cover to import ratio is in this case approximately 17%. This is in accordance with surveys on the topic. For example, Carse et al. (1980) found that firms that import or export and thus face terms-of-trade risk, only cover between 15% and 30% of their open positions.

Some caveats are in order here. First, the actual terms-of-trade variance may well be underestimated with our proxy used. If this is true, the calculated amount of forwards is too high as well. Secondly, the costs of forwards we used are to some degree arbitrary. They are, however, close to the actual transaction fees charged by banks but would not incorporate such items as information costs and fixed costs for setting up the appropriate institutions, letting alone deliberation costs. To the extent to which the actual costs are higher, our result overestimates the amount of forwards purchased. Thirdly, there is the issue of the degree of risk aversion with respect to wealth. In the literature there is no consensus on that parameter. We chose to set this parameter, $1 - \sigma$ in our model, to 1/2, which is a conservative choice in the sense that a broad range of publications support this choice. It also turns out that this particular parameter is the least influential in altering our results. Lastly, our result is to some extent related to the literature on international portfolio diversification, i.e. the home bias puzzle in equity holdings. Baxter and Jerman (1997) argue that, in order to explain actual portfolio holdings quantitatively, one needs to consider multiple sources of uncertainty. Recently, however, other contributions—see, for example, Obstfeld and Rogoff (2000) who consider trade costs as the relevant explanation for the observed home bias in equity holdings—have relied on a more parsimonious specification with only one source of uncertainty. Here, we follow the more parsimonious approach. The aforementioned qualifications notwithstanding, this numerical exercise recapitulates our analytical results and shows that the model is able to fit the actual data for reasonable parameter values.

Comparative Statics

There are a couple of interesting questions arising when changing the parameters. We begin with the terms-of-trade variance. If there is an exogenously induced increase in the variance of the foreign price, we observe a fall in the demand for forwards. At our calculated equilibrium point, we observe a decrease of 4.7% in demand for forwards if we increase the variance by 1%. This is in accordance with the intuition for our results. Risk-averse agents are not willing to give up the possibility to adjust themselves to a terms-of-trade shock. The greater the likelihood of a terms-of-trade shock, the more they have to be compensated for holding forward contracts.

Next, consider the costs of the forwards. If costs decrease, demand will increase. At the point of our interior solution a 1% decrease in the costs would induce a 16% rise in the demand for forward contracts.

Lastly, we look at the degree of risk aversion. A society which is more risk averse than another will demand less forward cover than the less risk-averse society. A 1% increase of the degree of risk aversion, i.e. a 1% fall in σ, reduces demand for forwards by 0.4%.

The comparative static results are summarized in Figure 3. An increase in the variance of p^Y, an increase in the costs χ and an increase in the degree of risk aversion will *ceteris paribus* decrease the demand for forward cover by shifting the schedule implied by (14) downwards. Note that in the case of changing costs χ, the capital schedule will also shift.

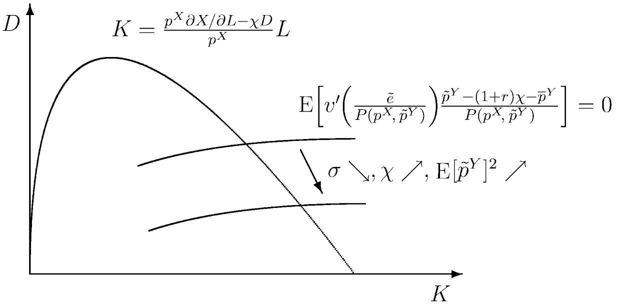

Figure 3. Comparative Static Results

Options

In order to give additional insights into the workings of our model, in this section we will examine what the optimal hedging behavior would be if the agents could buy options instead of forward contracts to insure against the uncertainty regarding the price of the foreign good. A (call) option, as opposed to a forward contract, does not oblige to buy the underlying asset (or commodity); instead the buyer can choose whether or not he will exercise his option. Keeping our notation, we can extend our model very easily to model an option instead of a forward contract by observing that, in the event $p_t^Y \leq \bar{p}^Y$, the buyer of that option would simply not exercise it. To model options, we only have to change the expenditure equation into

$$e_{t+1} = \begin{Bmatrix} (1+r_{t+1})(w_t - \chi D_t) \\ (1+r_{t+1})(w_t - \chi D_t) + (p_{t+1}^Y - \bar{p}^Y) D_{t+1} \end{Bmatrix} \quad \text{for all } p_{t+1}^Y \begin{Bmatrix} \leq \\ > \end{Bmatrix} \bar{p}^Y.$$

D_t now denotes the amount of options instead of forward contracts, the strike price being $\bar{p}^Y$. By buying one option for the price χ, an agent is entitled to buy one unit of good Y in the next period for the price $\bar{p}^Y$. The first-order condition (14) now becomes

$$\int_0^{\bar{p}^Y} v'\left(\frac{(1+r_{t+1})(w_t - \chi D_t)}{P(p^X, p_{t+1}^Y)}\right) \frac{-\chi(1+r_{t+1})}{P(p^X, p_{t+1}^Y)} dP^Y + \int_{\bar{p}^Y}^{\infty} v'\left(\frac{e_{t+1}}{P(p^X, p_{t+1}^Y)}\right) \frac{p_t^Y - \bar{p}^Y - \chi(1+r_{t+1})}{P(p^X, p_{t+1}^Y)} dP^Y = 0, \tag{21}$$

where P^Y is the cumulative density function of p_{t+1}^Y. Three results emerge for the steady state (see the Appendix).

THEOREM 4. *If options are costless, i.e.* $\chi = 0$, *the optimal amount is infinity,* $D = \infty$.

This is probably the most straightforward result. Rational agents, being offered a free lunch, will happily accept this. Here the free lunch comes as a free lottery ticket, without any risk of losing. We present this otherwise not very surprising result to make the structure of the decision problem clearer.

THEOREM 5. *If agents can choose between options and forwards at the same costs, they will always choose options.*

To facilitate the comparison between forwards and options, we present the second result. It constitutes, again, a standard property of the utility function of the agents. Forwards will always be dominated by options, as long as the price is the same for both.

These two theorems imply that we can replicate the real-world coexistence of options and forwards in our model. This necessitates that either forwards cost less or are more than unbiased (or both).

THEOREM 6. *Let transaction costs for options be given by* χ. *If options are unbiased, i.e.* $\mathrm{E}[\tilde{p}^Y] = \bar{p}^Y$, *agents will demand a positive number of options.*

Our last result highlights again the difference between forwards and options. In contrast to forward contracts there exists a positive demand, depending on the price χ, of "unbiased options," i.e. options that have a strike price that equals the expected value of the price in the next period.

5. Conclusion

One largely debated issue in international economics is the question whether or not volatility in exchange rates and terms of trade depresses trade levels. There is an extensive literature on that question, both theoretical and empirical. The main body of the theoretical literature claims that terms-of-trade and/or exchange rate uncertainty does not matter as long as well-developed forward and futures markets exist. This literature further predicts that agents fully hedge the existing risks. The empirical work done in this field fails to unambiguously support these findings.

We model a small open economy that is subject to terms-of-trade risk originating entirely from abroad. Agents can buy forward contracts to insure against this uncertainty but can adjust consumption bundles after terms of trade have realized. This small departure from the standard assumption in the hedging literature where consumption cannot be adjusted after resolution of uncertainty implies that forward contracts turn into an asset. When forward contracts are unbiased, there is no demand for terms-of-trade insurance, a direct effect of the convexity of the indirect utility function with respect to prices. Risk aversion with respect to consumption levels and expenditure levels is not a sufficient motive to buy forwards. We derive conditions under which, on part of the risk averters, a positive demand for forwards exists. Again, this demand does not stem from hedging but purely from investment motives.

We calibrate our model with data for Germany to obtain numerical solutions. The equilibrium amount of forwards contracted in relation to the equilibrium amount of imports closely resembles the empirical observed values, thus providing a rationale for the apparent underhedging of domestic agents against price level and/or exchange rate uncertainty. The reason for low hedging lies again in the asset-nature of forwards: as returns for forwards should be lower than returns for, e.g. capital, few futures will be held and hedging is low. We also showed that options, in contrast to forwards, will be bought as means of insurance. At unbiased prices, options strictly dominate forward contracts. This may help to explain why the market for options has grown exponentially over the last decade or so.

The main contribution of our analysis, however, is that the "price-convexity" effect should be incorporated in the existing models, which could be achieved by giving up the assumption that all plans are irrevocably made in the period which precedes the resolution of the uncertainty. This should dramatically alter the strong theoretical predictions of this literature with respect to forward markets and should thus provide a better understanding of the effects at work here. Since forwards are unattractive and options perhaps too expensive, our analysis may also provide an additional argument *in favor* of international capital flows, and hence capital account liberalization, as a means of insuring the economy.

Our work can be extended in some promising ways. First, to understand the implications of covariance effects so often at work in the hedging process, money and thus a nominal exchange rate could be brought into the model. This would also allow a comparison between our modeling approach and the existing literature that has proceeded with considering multiple sources of risk. Another interesting extension would be to explicitly study the effect of heterogeneity in risk aversion. This would allow to endogenize the forward price $\overline{p}^{Y}$ and thereby to confirm (as we would expect) that returns on forwards as assets are low. This would strengthen our explanation that trade coverage is low because forwards are assets.

Appendix

All further computations are contained in a referees' appendix, which is available at www.waelde.com/publications.html.

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Notes

1. In contrast to the majority of the literature on that topic, households demand forwards, not firms. This, however, simply follows from the general-equilibrium set-up we use. Firms are owned by the households, who look "through" them. A similar argument is made in Bacchetta and van Wincoop (1998, p. 18).
2. If, in contrast, D_t represented options, domestic agents would not be obliged to buy and thus only draw on the contract in favorable situations. This will be analyzed in section 4.
3. We are grateful to one referee who pointed out that our set-up is similar to an endowment economy. The endowment is given by the wage w_t and agents decide whether to transfer this endowment into the next period by holding capital or buying forward contracts.
4. In the literature, actuarially fair refers to a situation where the expected pay-off of an insurance is equal to the insurance premium (Dixit, 1990, p. 124, or Kreps, 1990, p. 92). Unbiasedness usually describes a (statistical) property of the forward price, i.e. $E[p^Y] = \overline{p}^Y$ (Zilcha and Broll, 1992, p. 475, or Viaene and Zilcha, 1998, p. 594). As we do not explicitly model how the forward price $\overline{p}^Y$ is determined, we use the expression "unbiasedness." Note, however, that the two concepts would be identical if we assumed that the forward price is the outcome of competition among perfectly competitive firms and χ are opportunity cost of buyers of insurance (e.g. shipping cost) and not risk premia for the insurer.
5. All numerical results were obtained by using Mathematica. The files are available on request.
6. See, *inter alia*, Ethier (1973), Benninga et al. (1985), Kawai and Zilcha (1986), Eldor and Zilcha (1987), Viaene and de Vries (1992), Zilcha and Broll (1992), Viaene and Zilcha (1998), and Adam-Müller (2000).
7. There are a few papers that discuss the theoretical possibility of a different timing structure, an example being Perée and Steinherr (1989). We are, however, not aware of any work that explicitly models this.
8. Strictly speaking, we should write $(p^Y)^\alpha$. To simplify notation, we use p_Y^α and deviate from our convention of indicating the type of the good with superscripts.
9. Think of a firm that has to hire expertise to contract such cover and thus may have substantial costs. In terms of transfers, the χD's, think of margin requirements.
10. This follows from $(1 - 0.025)^{30} \approx 0.46$. Hence 46% of the capital stock remains and 54% is lost after 30 years of constant annual depreciation of 2.5%.
11. As stated in the model section, $\overline{p}_Y$ is exogenously given by international markets. We use this equation to find a plausible value for $\overline{p}_Y$. It does not mean that $\overline{p}_Y$ is endogenous in our model.
12. The Black–Scholes formula relies on log-normality of prices. Even in international macro this assumption is often used; see, for example, Obstfeld and Rogoff (1998).
13. We use *R* and the function *fitdistr*, which is included in the MASS package.